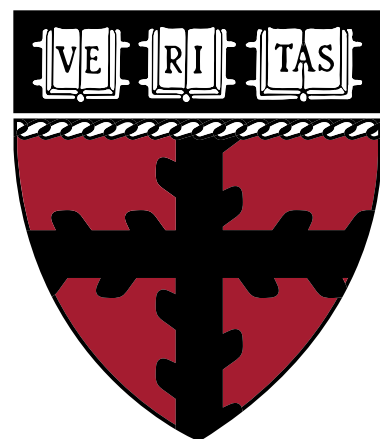


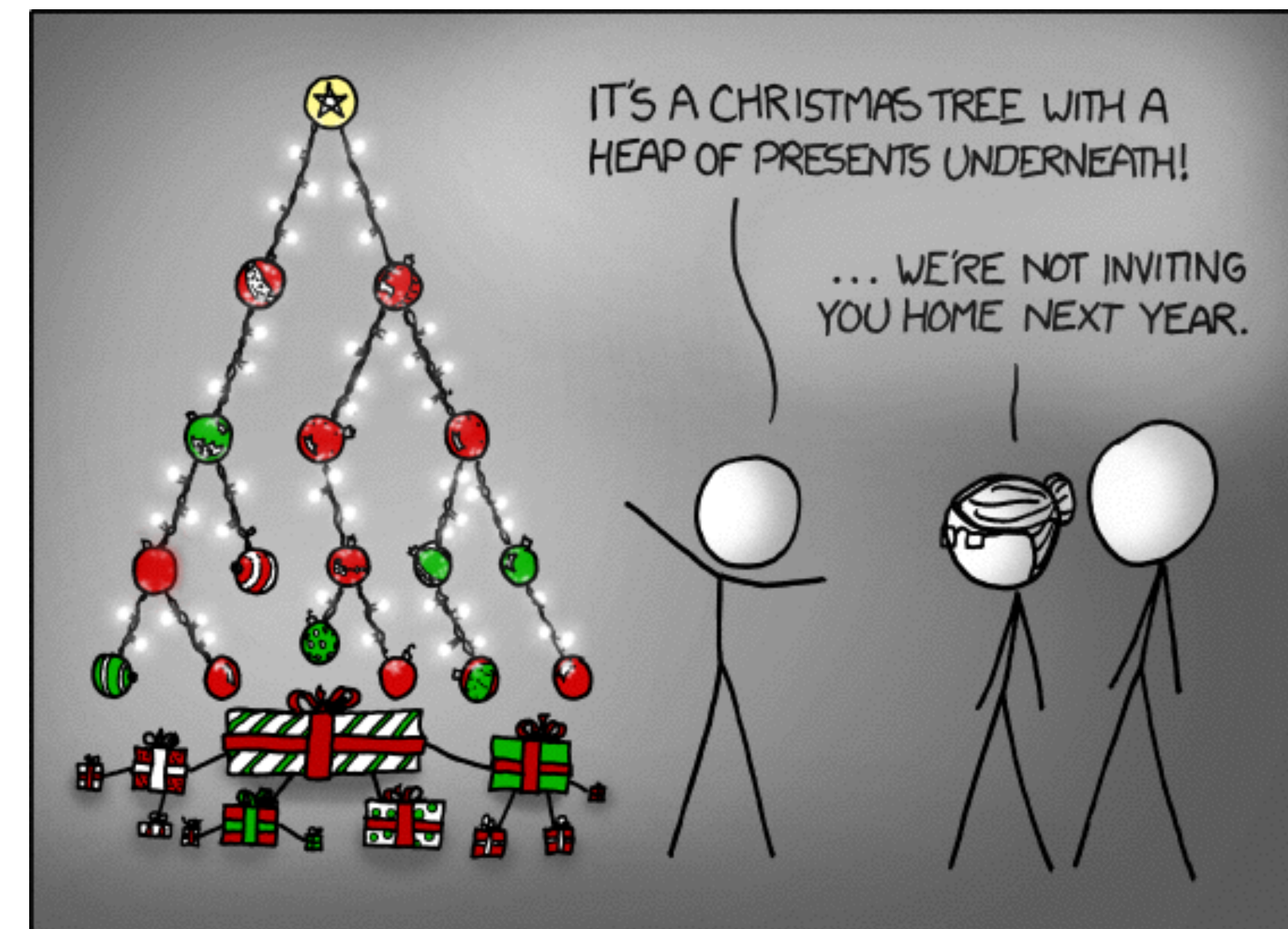
# CS171 Visualization

Alexander Lex  
[alex@seas.harvard.edu](mailto:alex@seas.harvard.edu)

## Graphs



**HARVARD**  
School of Engineering  
and Applied Sciences



[xkcd]

# This Week

Reading: VAD, Chapters 9

Lecture 12: Text & Documents

Sections: D3 and JS Design Guidelines. HW1 Review.

Updates

Design Studio moved to Tuesday after Spring-Break

HW 4 consists of “only” the project proposal

# Design Exercise

Data & Use Case by Augusto Sandoval

# Student question: How to show this data?

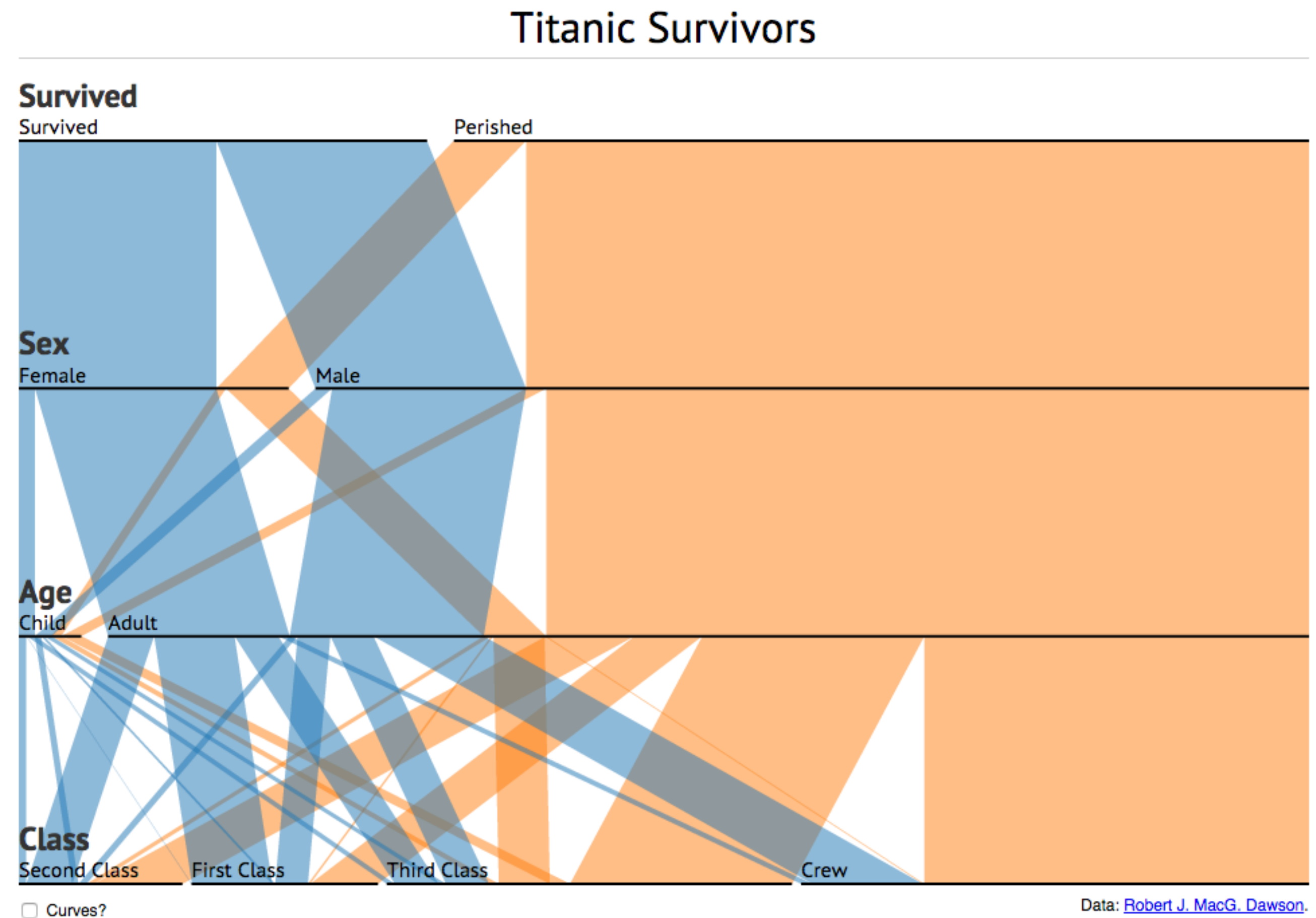
- ID
- Gender
- High School Type
- Degree
- Year of Admission
- GPA
- GPA z-score

Id	gender	e of High Sch	Degree	Year of admission	GPA	GPA zcore by Degree
nNYliG0HwFTjdFNks1	M	Private	journalism	2012	0	-6.384275
eT+GiwP0Bm4QJ3cZ	M	Public	engineering	2008	0.1888888888	-5.895198
ryP0ztCYW26aMdUq	M	Private	Laws	2004	0	-5.730906
S034U/Z7g9IQXTuU9	M	Private	Arts	2012	0.2666666666	-5.028417
9Z16bxKDeVDbGiiD8	F	Private	Arts	2012	1.6666666667	-2.331488
4EI03gz2P0m1wkWAl	M	Private	engineering	2013	0.6	-4.865675
lc+5vQGcTRba7j/vTxf	M	Private	engineering	2012	0.8444444444	-4.363712
9Rqf6BCleCiywUVdG	F	Private	history	2012	0.76	-4.033634
iltFP+NRF/yVvW2yLk	M	Private	agronomy	2012	0.84	-3.64938
nKiM2cmilt5hXdGQp	M	Private	engineering	2009	1	-4.000308
2s9GWUyTNMwyl8al	M	Public	theology	2008	0.9111111111	-3.627317
ug3OgnYirUcEUImXtJ	F	Private	nursing	2011	1.7333333333	-4.0063
Sm+XS3+8amJFzowVj	M	Private	theology	2013	0.9833333333	-3.507009
jrwfkQmu9YDTzWdEl	F	Public	nursing	2010	1.85	-3.495274
Jv6VtB+mIVY30ZYR4l	M	Private	business	2012	1.3166666667	-3.428196
szP2BH1uaYrsk3w9JC	F	Private	COLLEGE	2012	1	-3.345934
ellKHcnNQ0fW9jxR94	F	Private	Laws	2013	1.2533333333	-3.256467
qdY8dwFW0CxGdqUl	F	Private	nursing	2008	1.9333333333	-3.250402



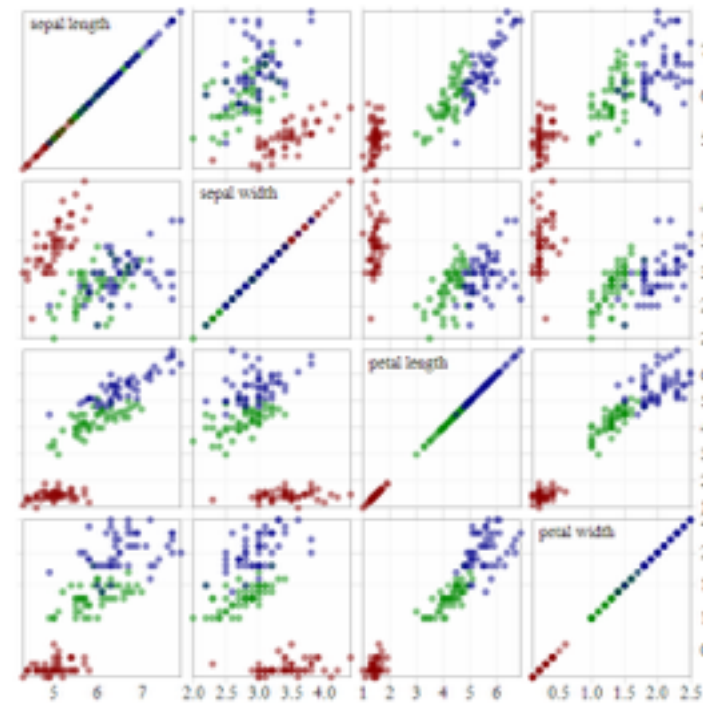
# Visualizing Categorical Data

Example:  
Parallel Sets



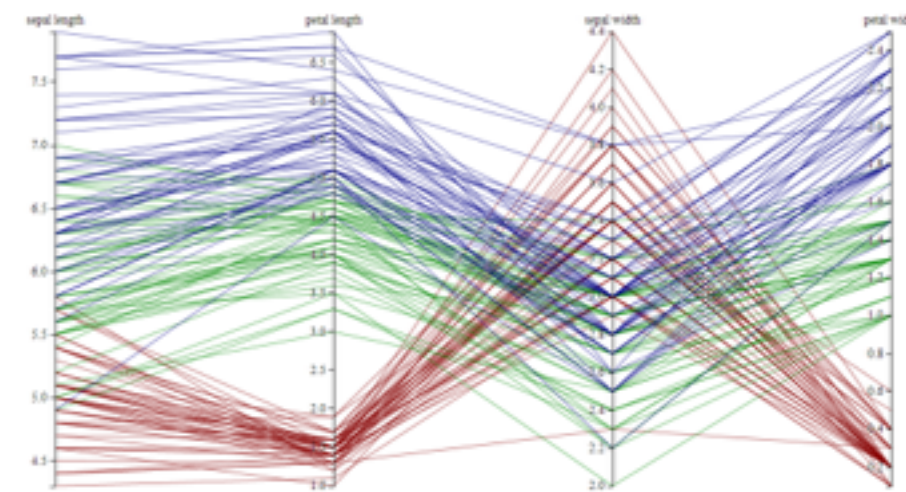
# Last Week: Highdimensional Data

# Analytic Component



## Scatterplot Matrices

[Bostock]



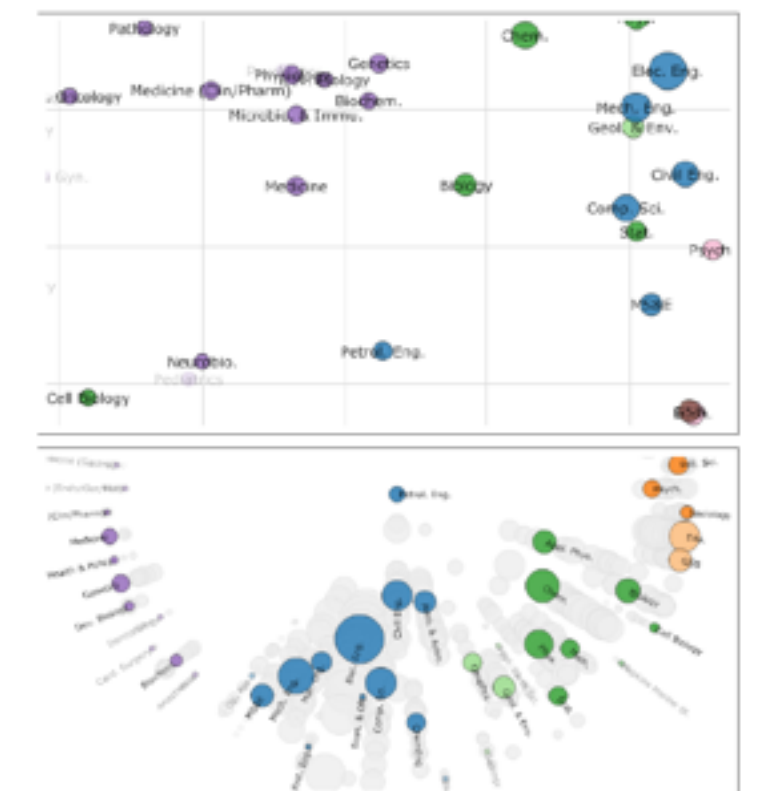
## Parallel Coordinates [Bostock]

## Pixel-based visualizations / heat maps



## Multidimensional Scaling

[Doerk 2011]



**[Chuang 2012]**

## no / little analytics

# strong analytics component

# Geometric Methods

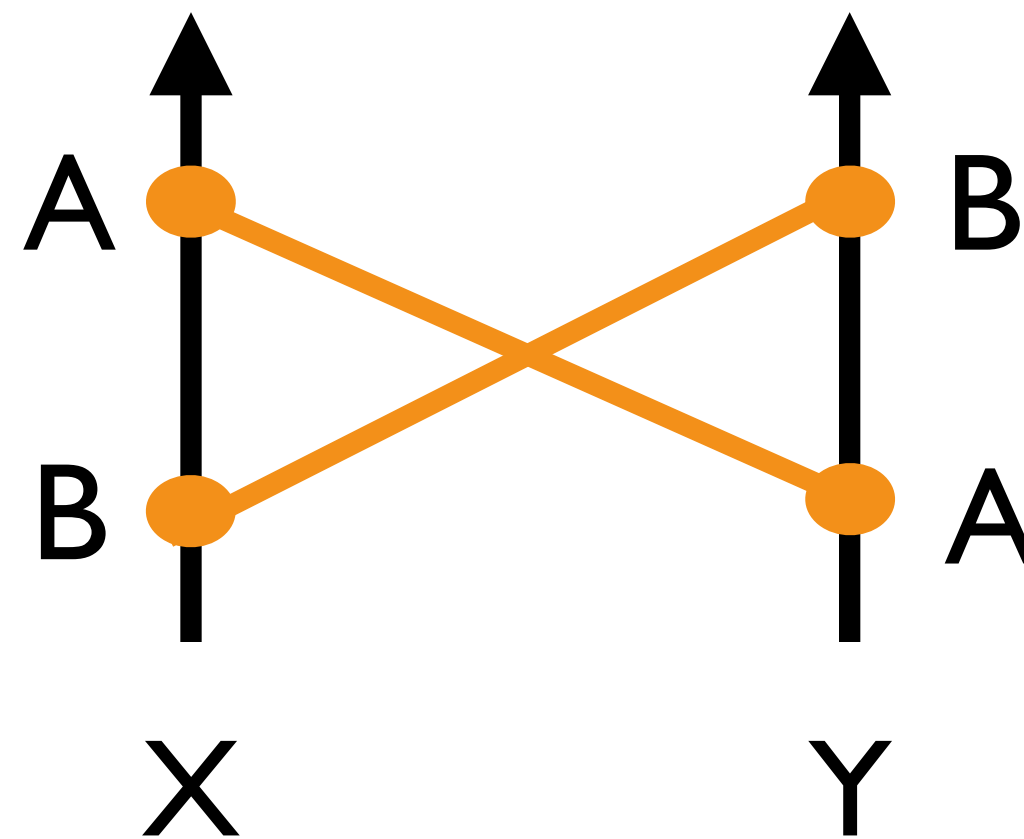
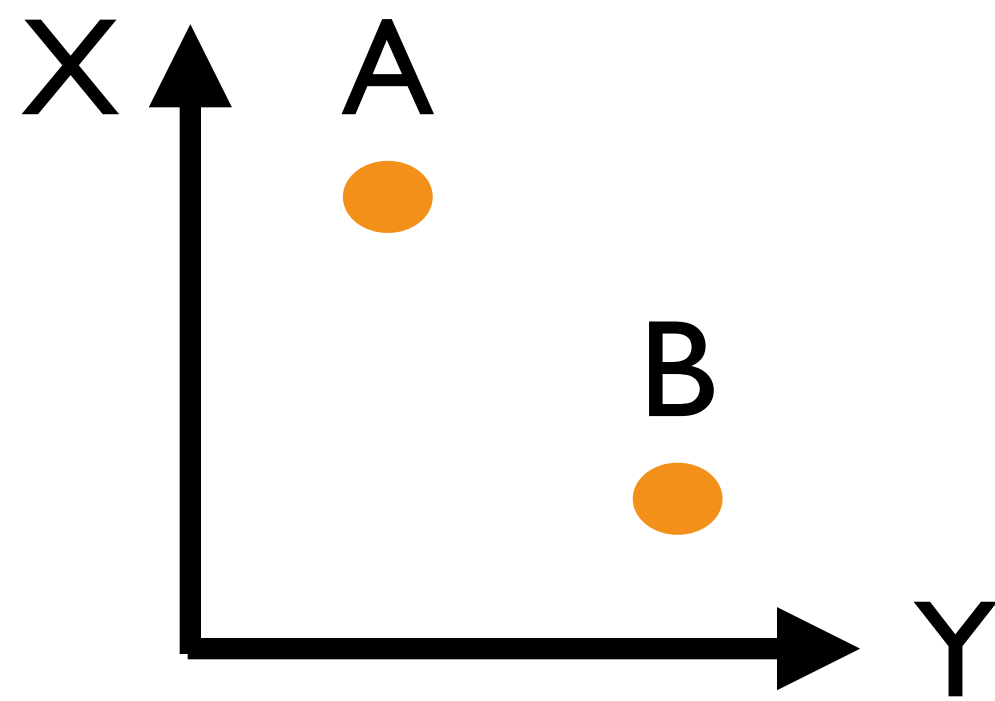


# Parallel Coordinates (PC)

Inselberg 1985

Axes represent attributes

Lines connecting axes represent items



# Parallel Coordinates

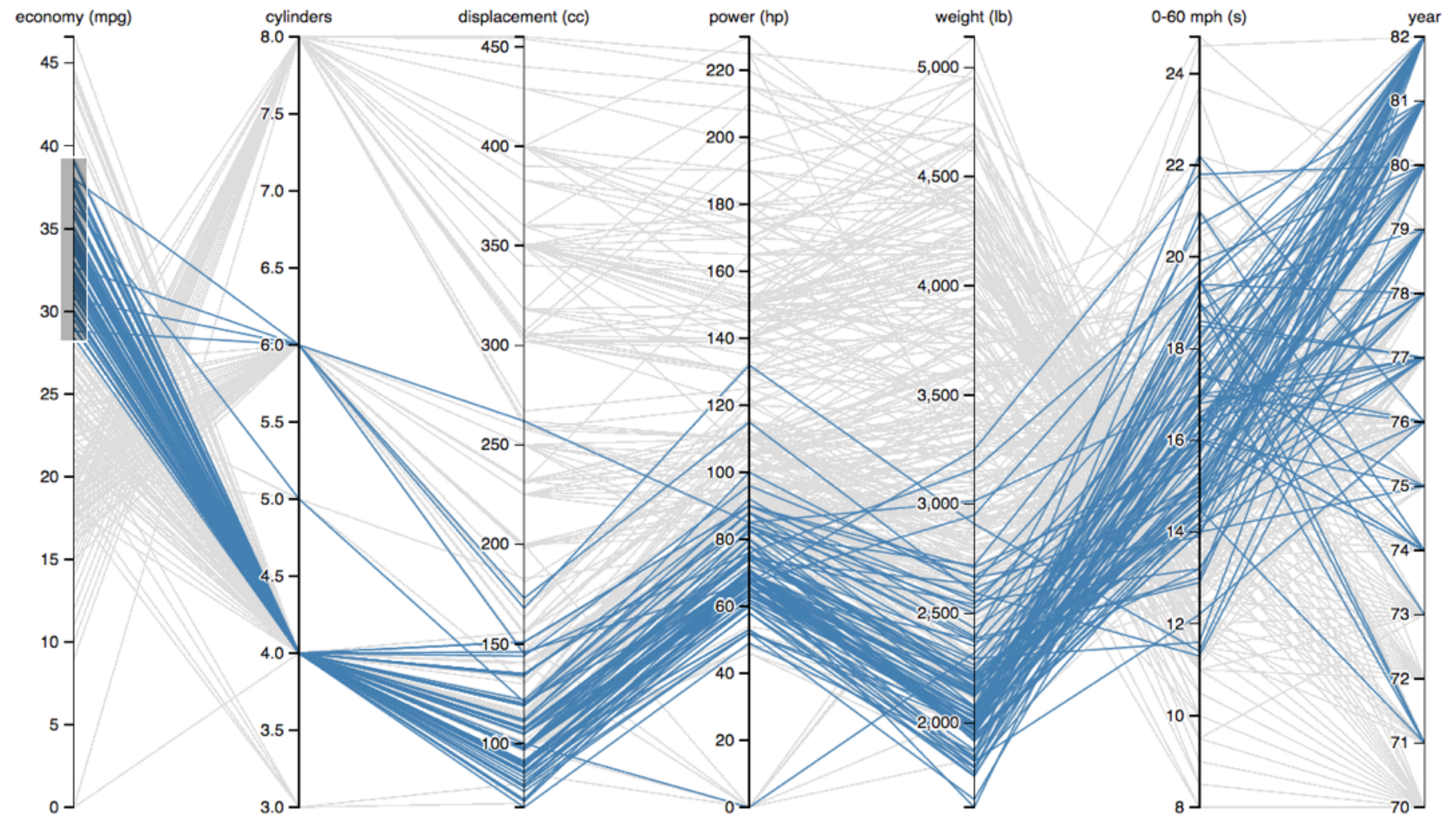
Each axis represents dimension

Lines connecting axis represent records

Suitable for

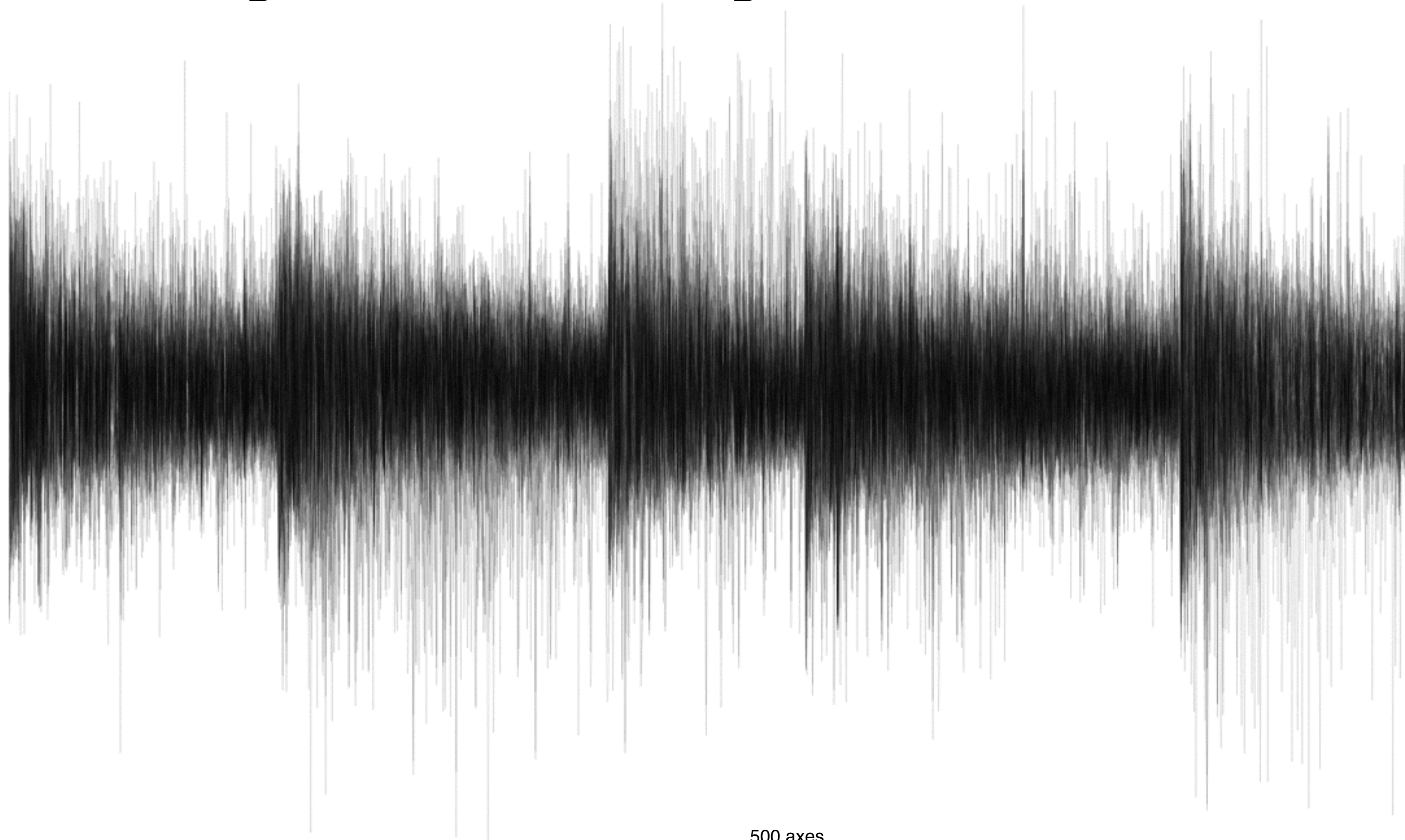
- all tabular data types

- heterogeneous data





# PC Limitation: Scalability to Many Dimensions



500 axes



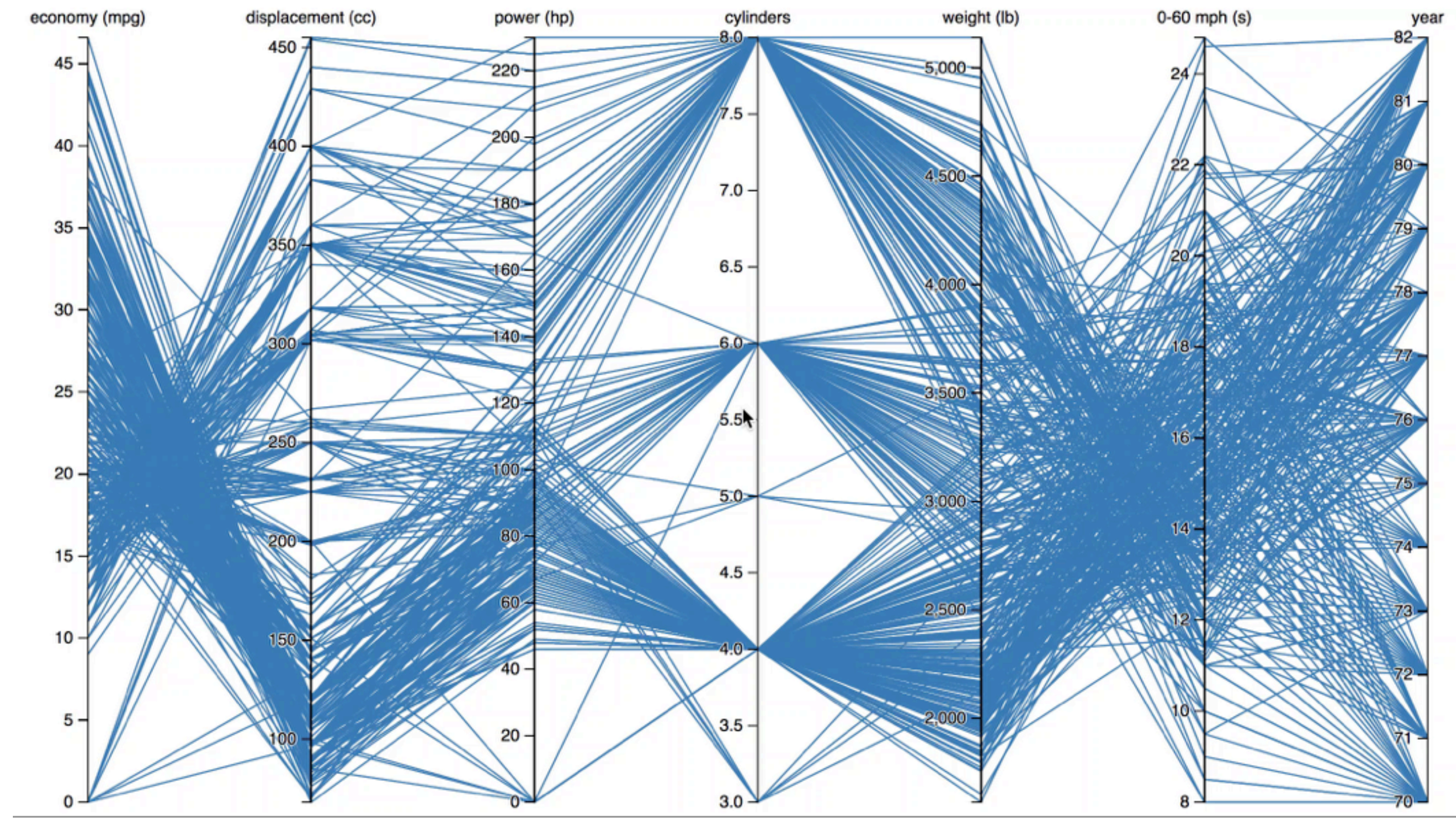
# PC Limitations

Correlations only between adjacent axes

Solution: Interaction

Brushing

Let user change order





# Parallel Coordinates

Shows primarily relationships between adjacent axis

Limited scalability (~50 dimensions, ~1-5k records)

Transparency of lines

Interaction is crucial

Axis reordering

Brushing

Filtering

**Algorithmic support:**

Choosing dimensions

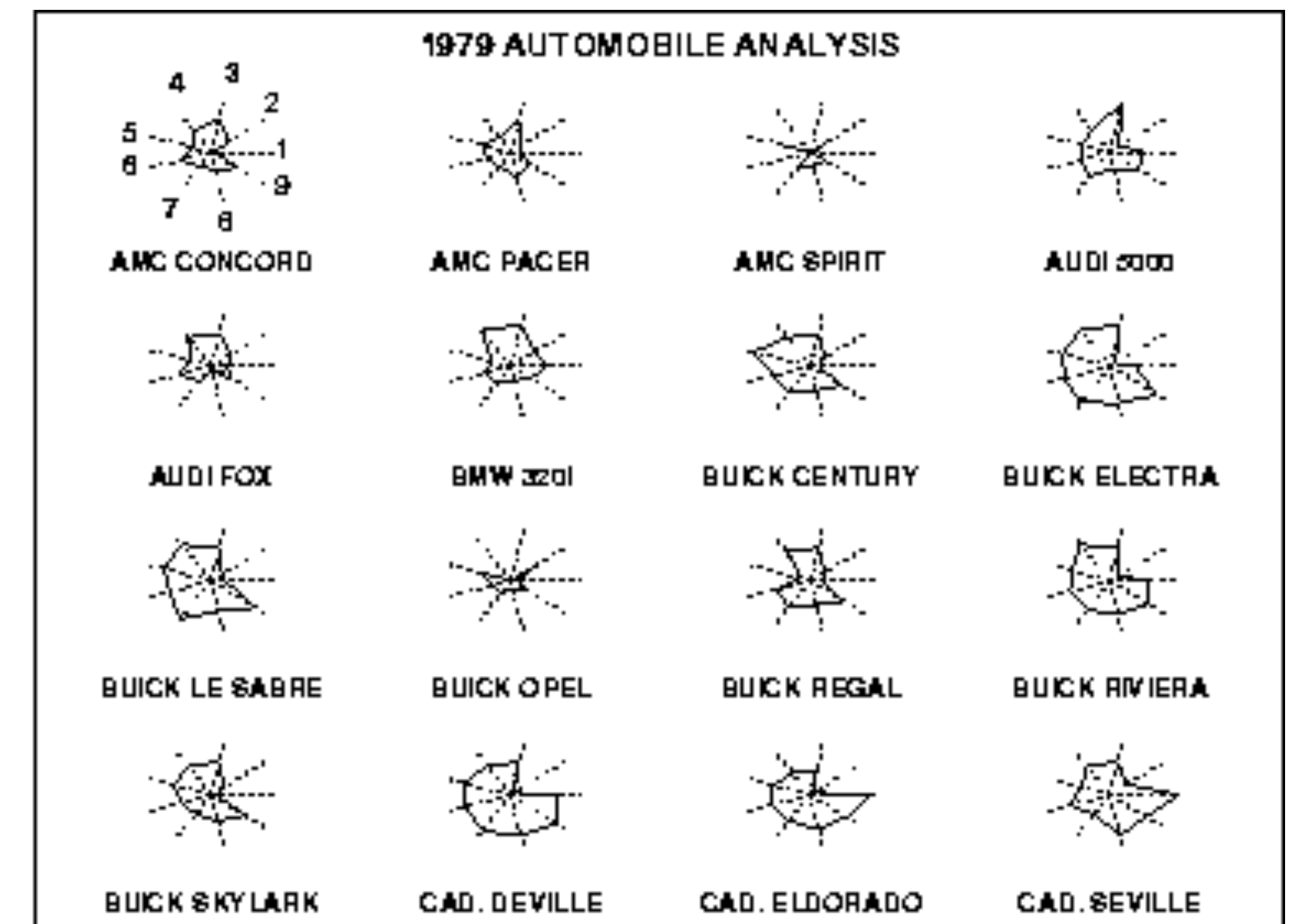
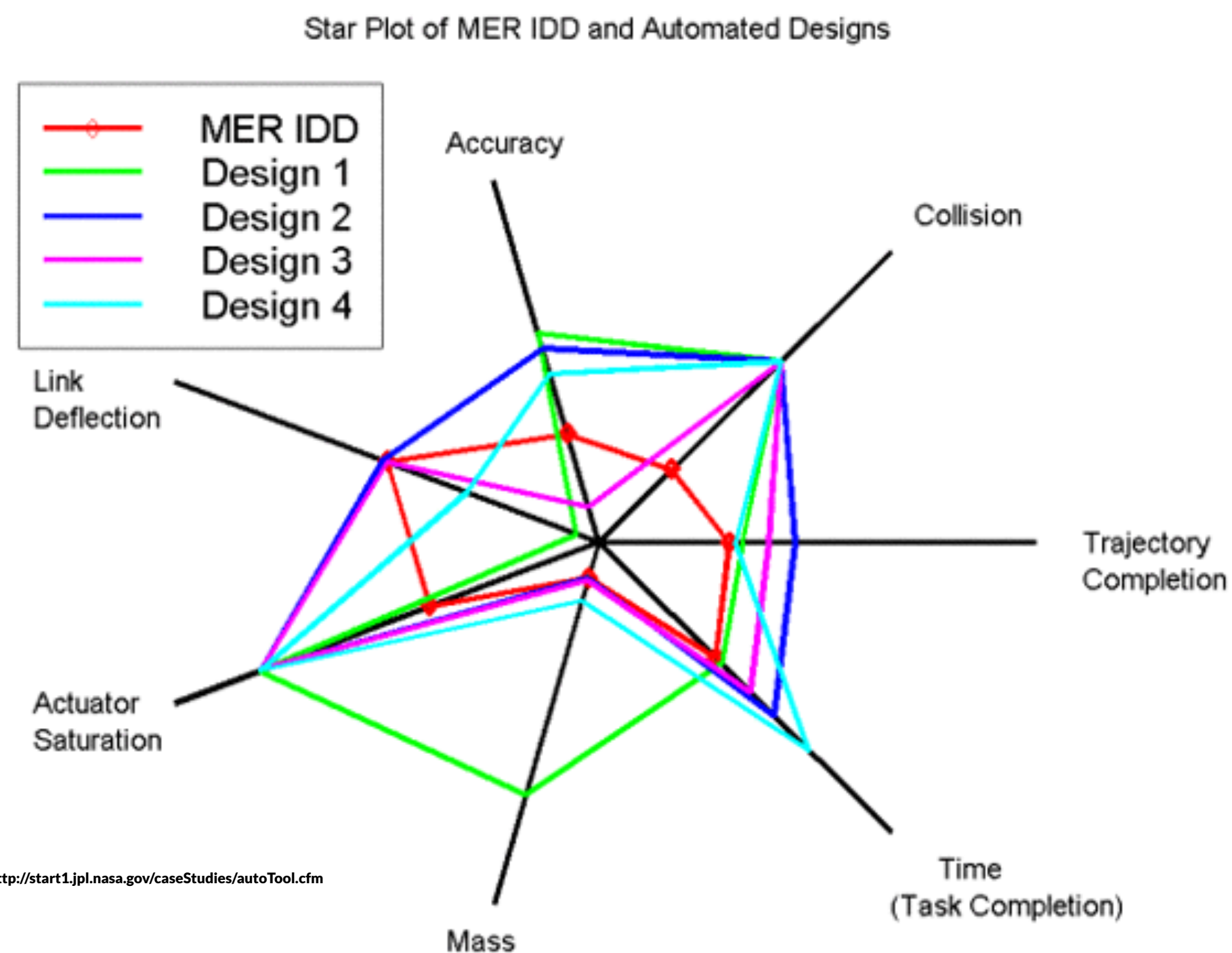
Choosing order

Clustering & aggregating records

# Star Plot

[Coekin1969]

Similar to parallel coordinates  
Radiate from a common origin



<http://www.itl.nist.gov/div898/handbook/eda/section3/starplot.htm>

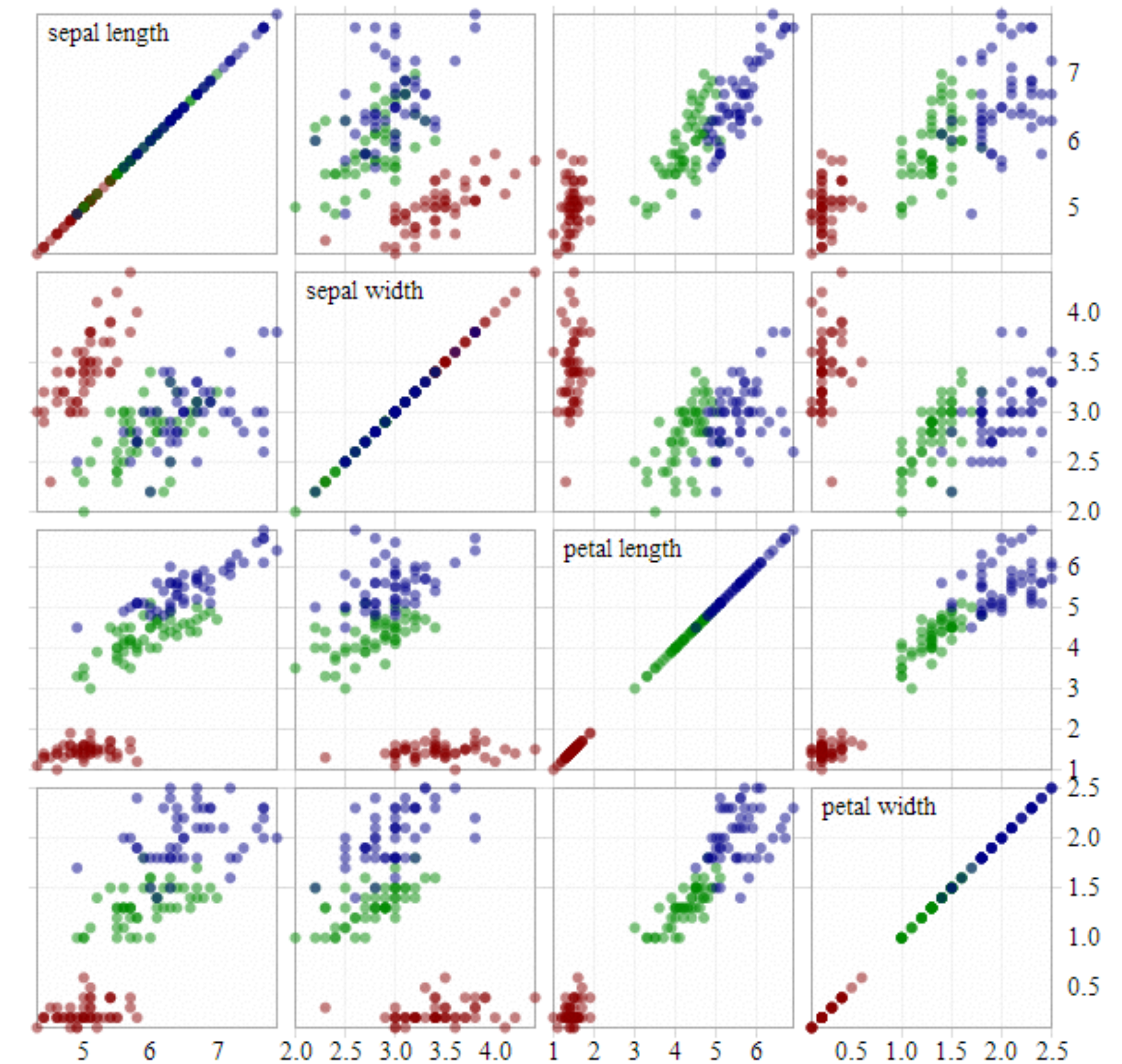
<http://blocks.org/kevinschaul/raw/8833989/>

# Scatterplot Matrices (SPLOM)

Matrix of size  $d \times d$

Each row/column is one dimension

Each cell plots a scatterplot of two dimensions



# Scatterplot Matrices

Limited scalability (~20 dimensions, ~500-1k records)

Brushing is important

Often combined with “Focus Scatterplot” as F+C technique

**Algorithmic approaches:**

Clustering & aggregating records

Choosing dimensions

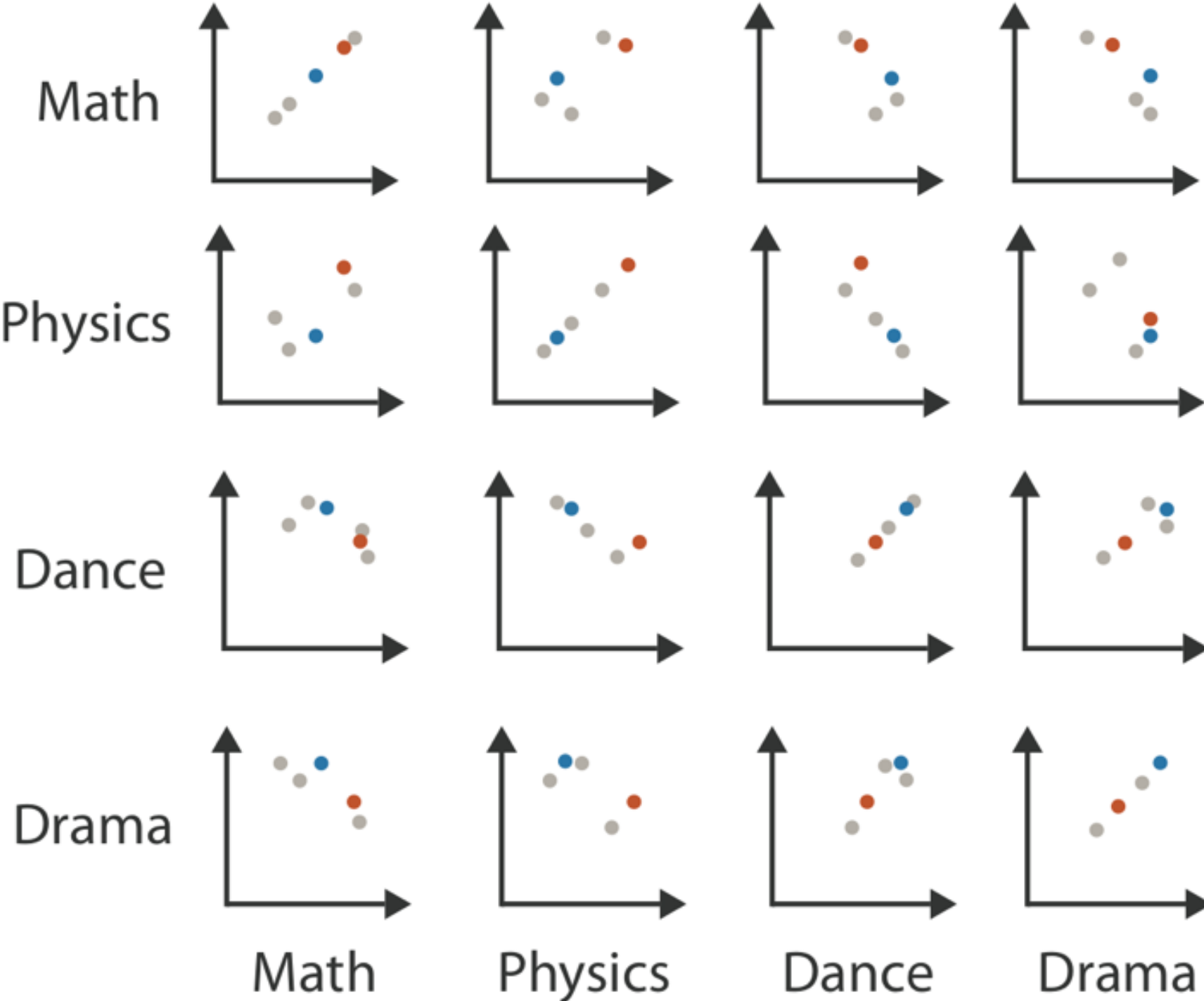
Choosing order



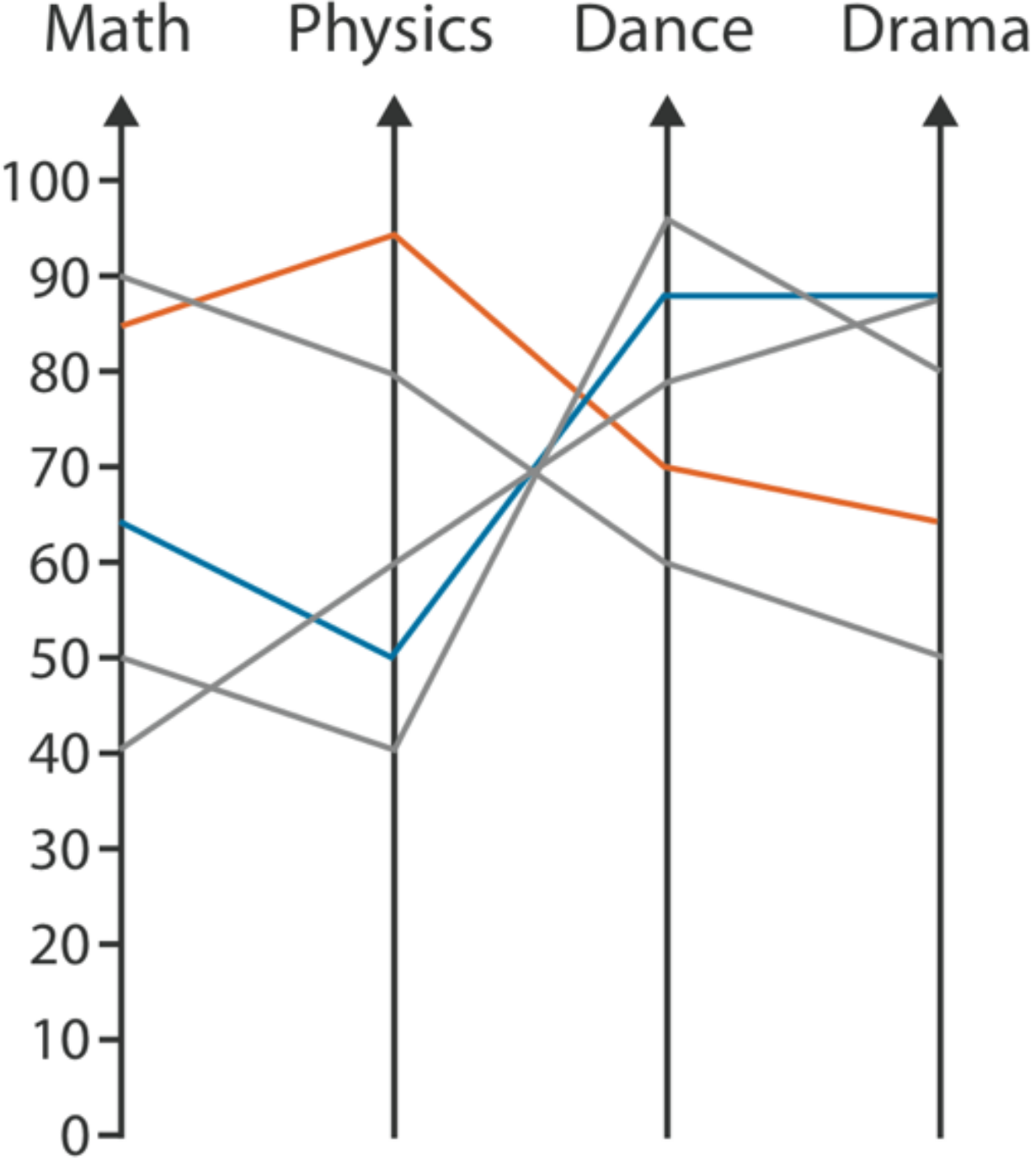
Table

	Math	Physics	Dance	Drama
Math	85	95	70	65
Physics	90	80	60	50
Dance	65	50	90	90
Drama	50	40	95	80
	40	60	80	90

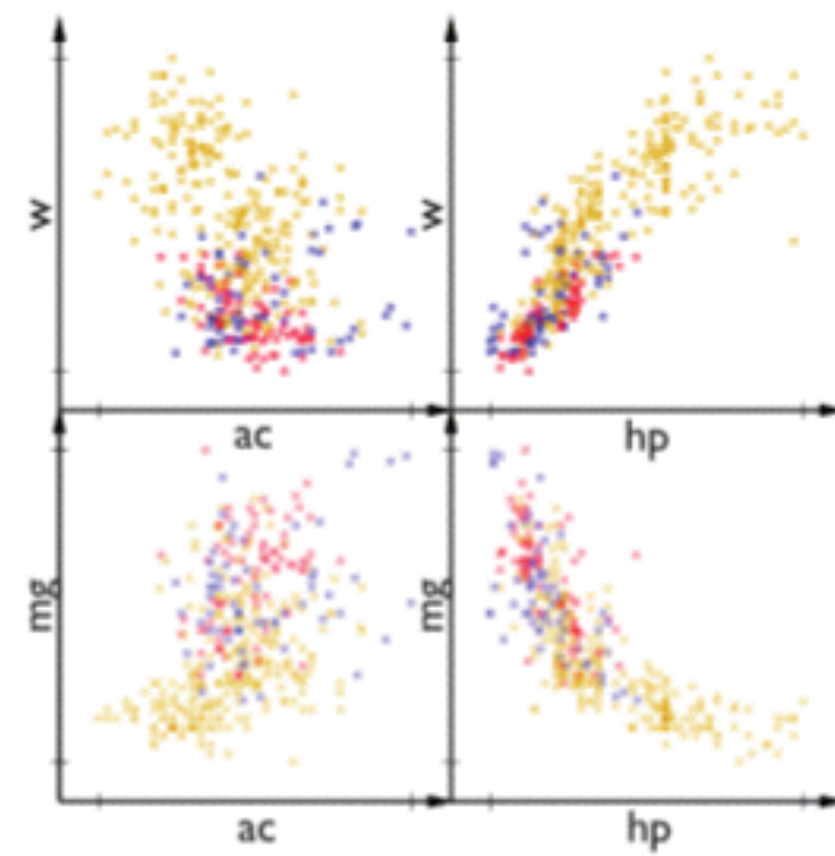
Scatterplot Matrix



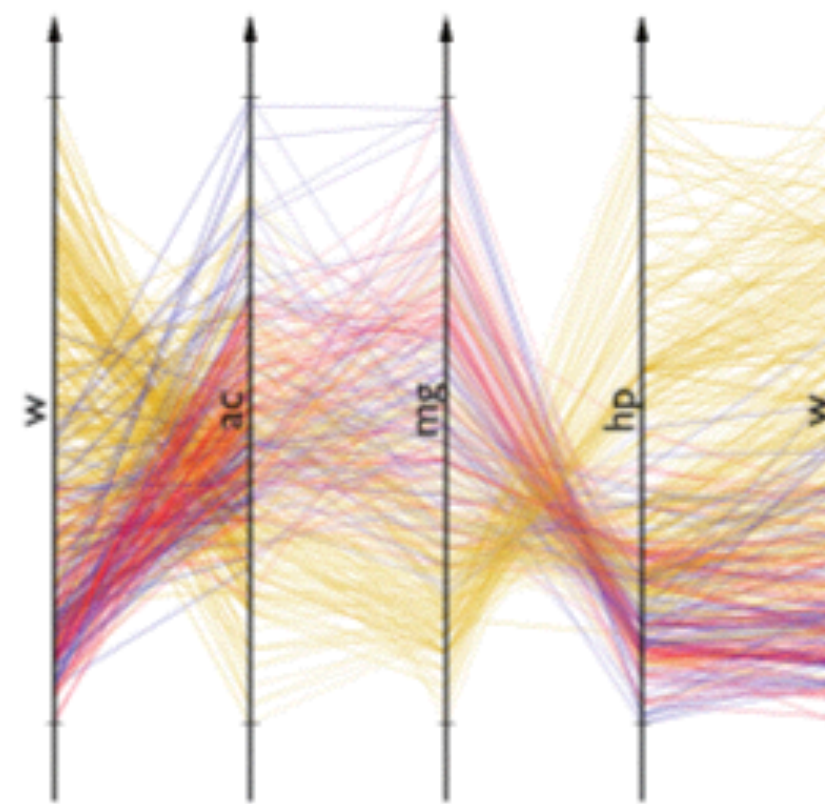
Parallel Coordinates



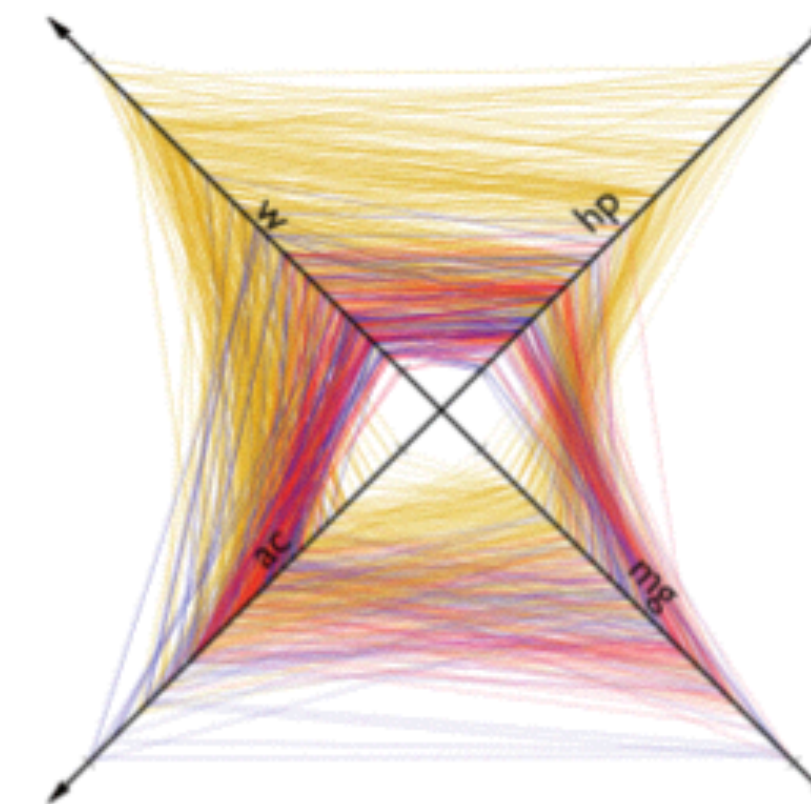
# Flexible Linked Axes (FLINA)



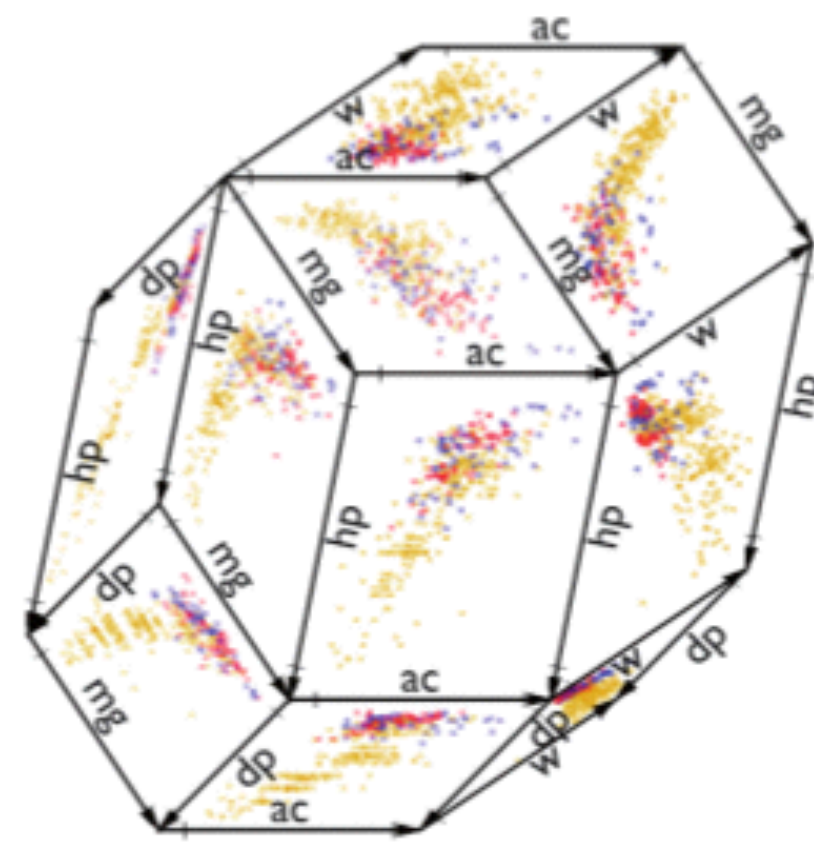
(a) scatterplots



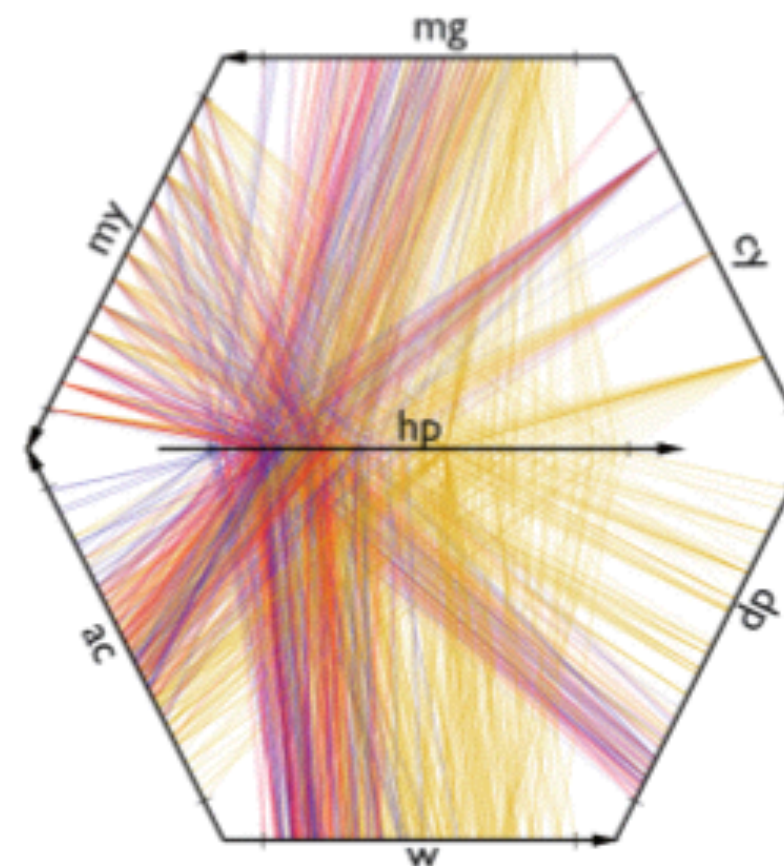
(b) Parallel Coordinates Plot



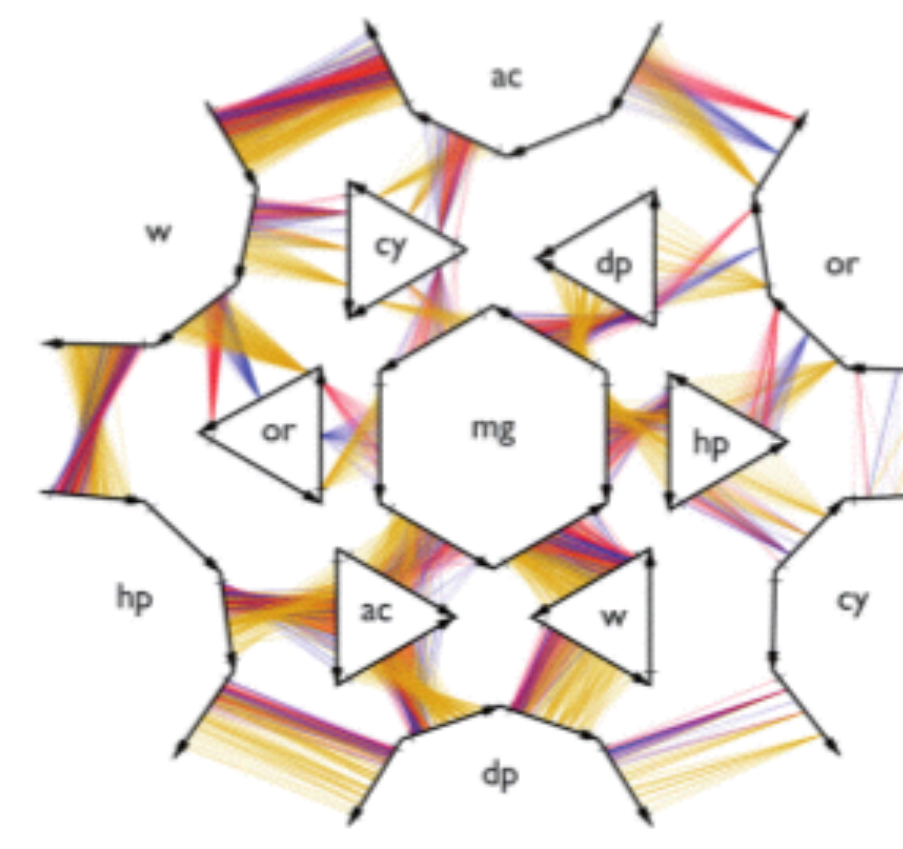
(c) radar chart



(d) Hyperbox



(e) Time Wheel



(f) Many-to-many PCP



# Data Reduction

## Sampling

**Don't show every element, show a (random) subset**

**Efficient for large dataset**

**Apply only for display purposes**

**Outlier-preserving approaches**

## Filtering

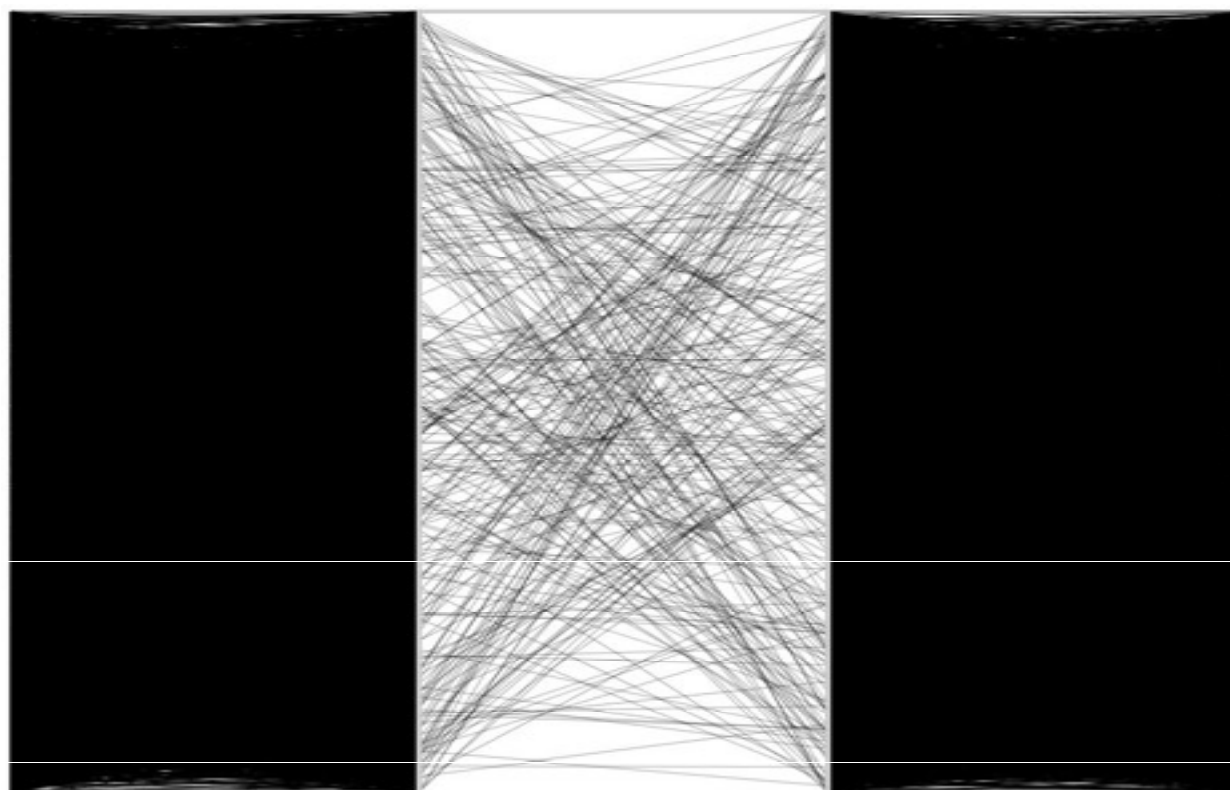
**Define criteria to remove data, e.g.,**

**minimum variability**

**> / < / = specific value for one dimension**

**consistency in replicates, ...**

**Can be interactive, combined with sampling**



[Ellis & Dix, 2006]

# Pixel Based Methods



# Pixel Based Displays

Each cell is a “pixel”, value encoded in color / value

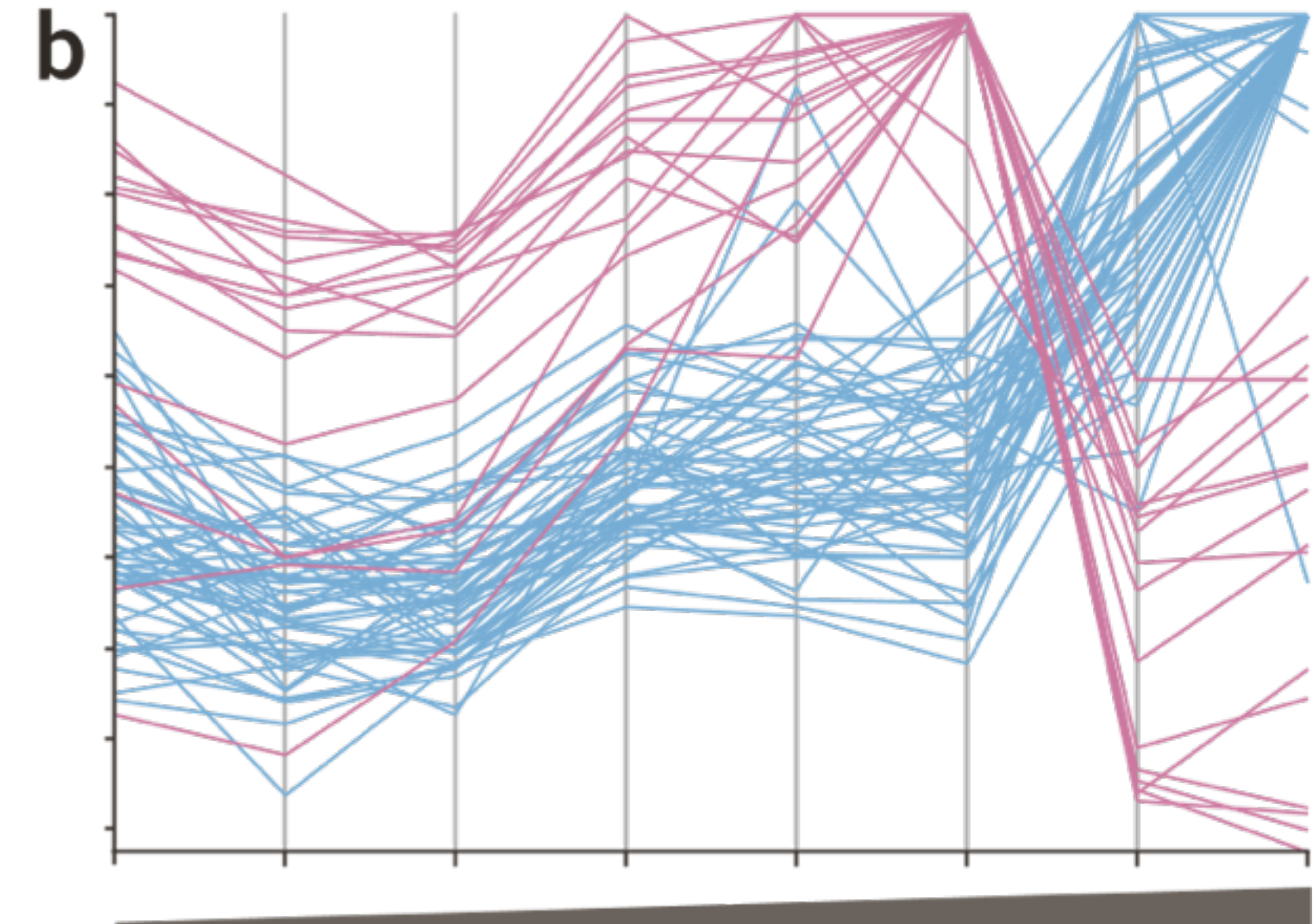
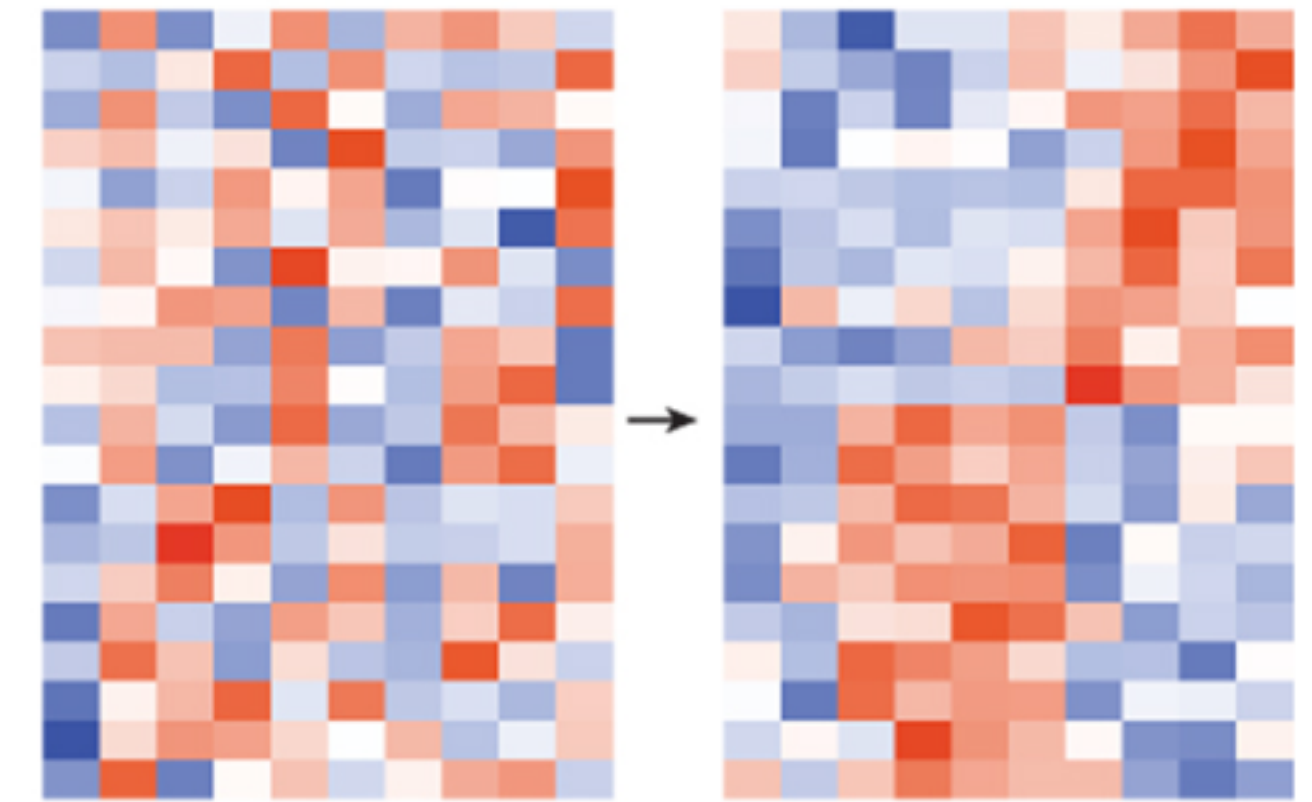
Meaning derived from ordering

If no ordering inherent, clustering is used

Scalable – 1 px per item

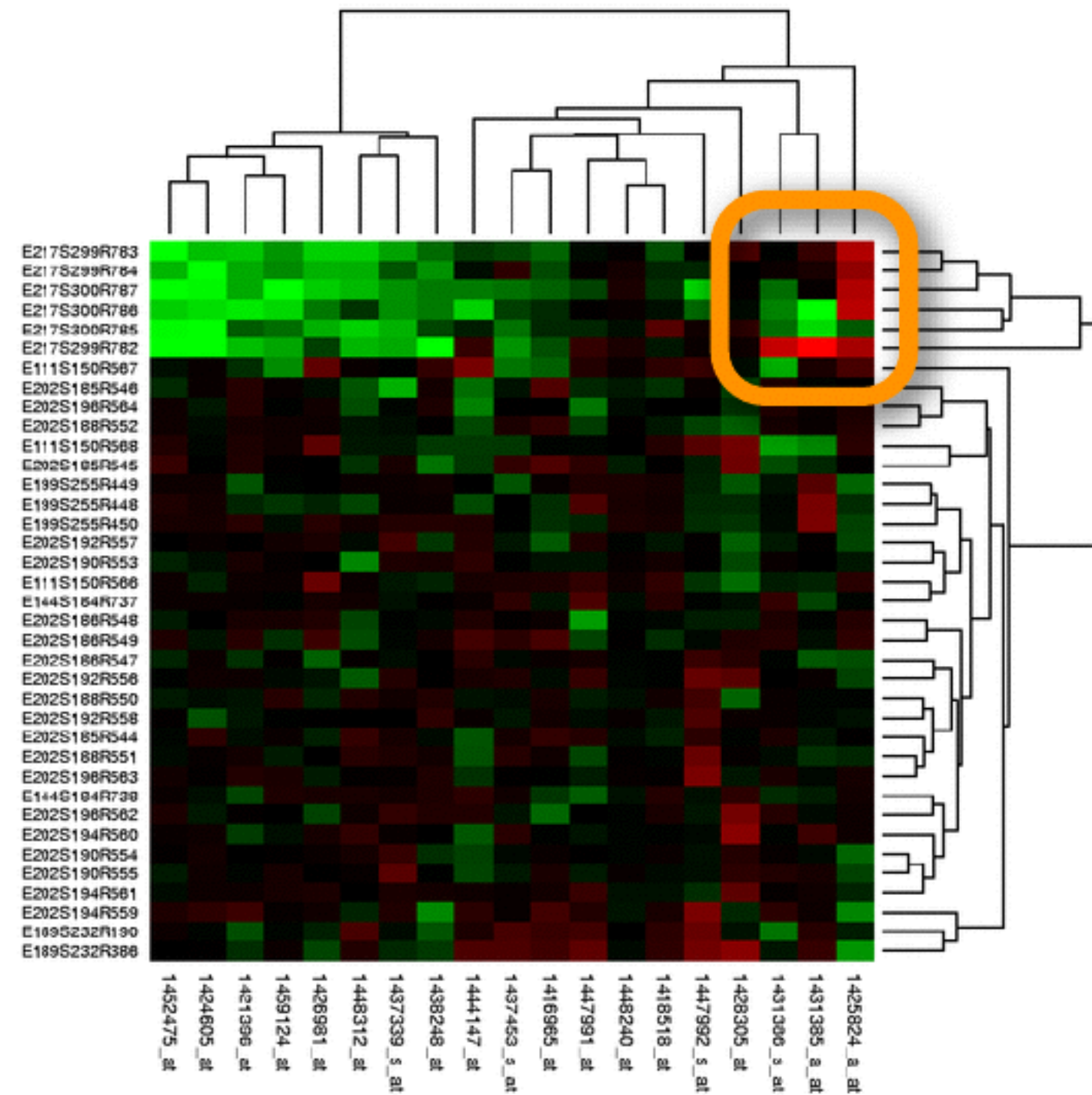
Good for homogeneous data

same scale & type

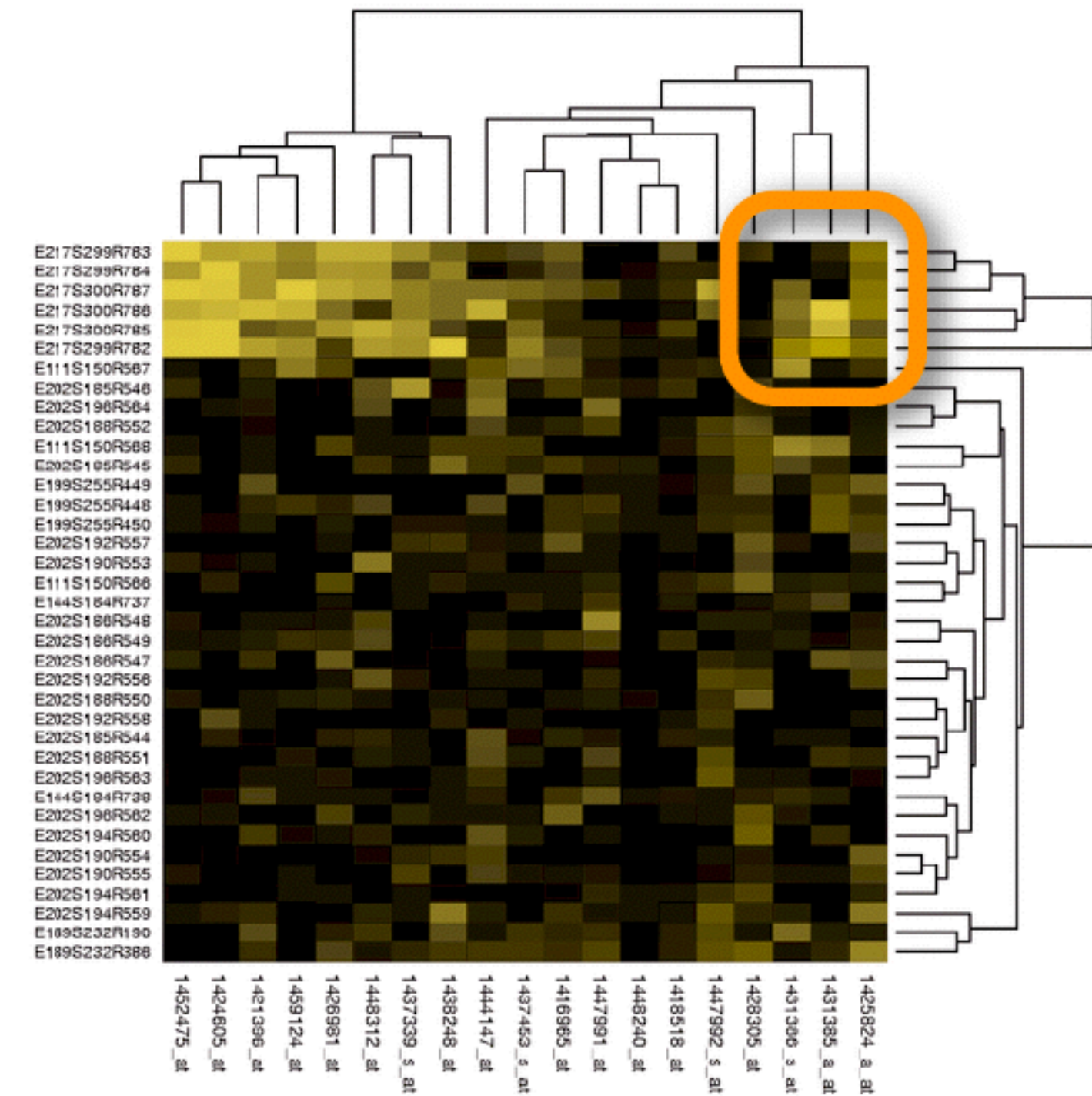




# Bad Color Mapping

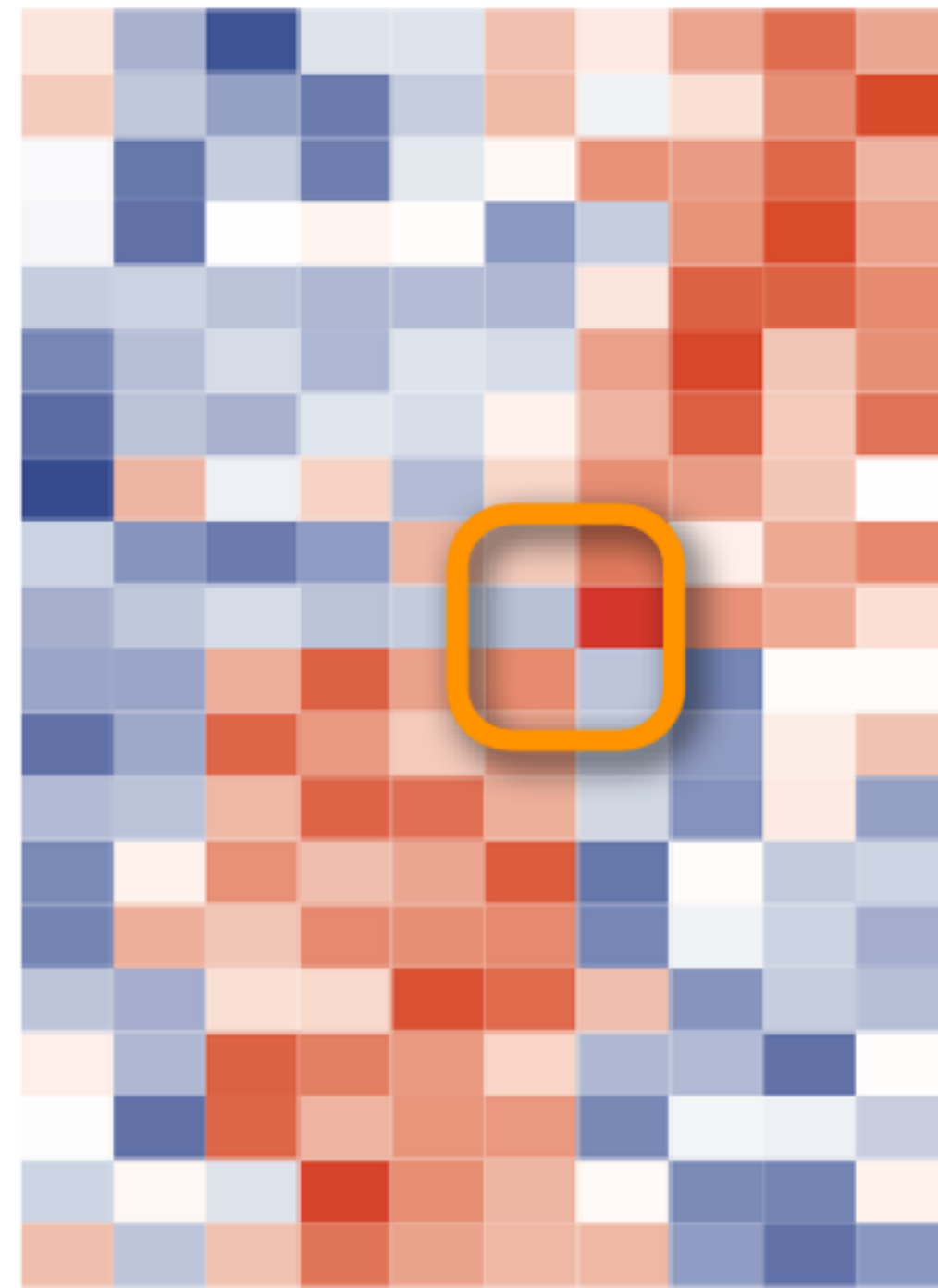


Normal Vision

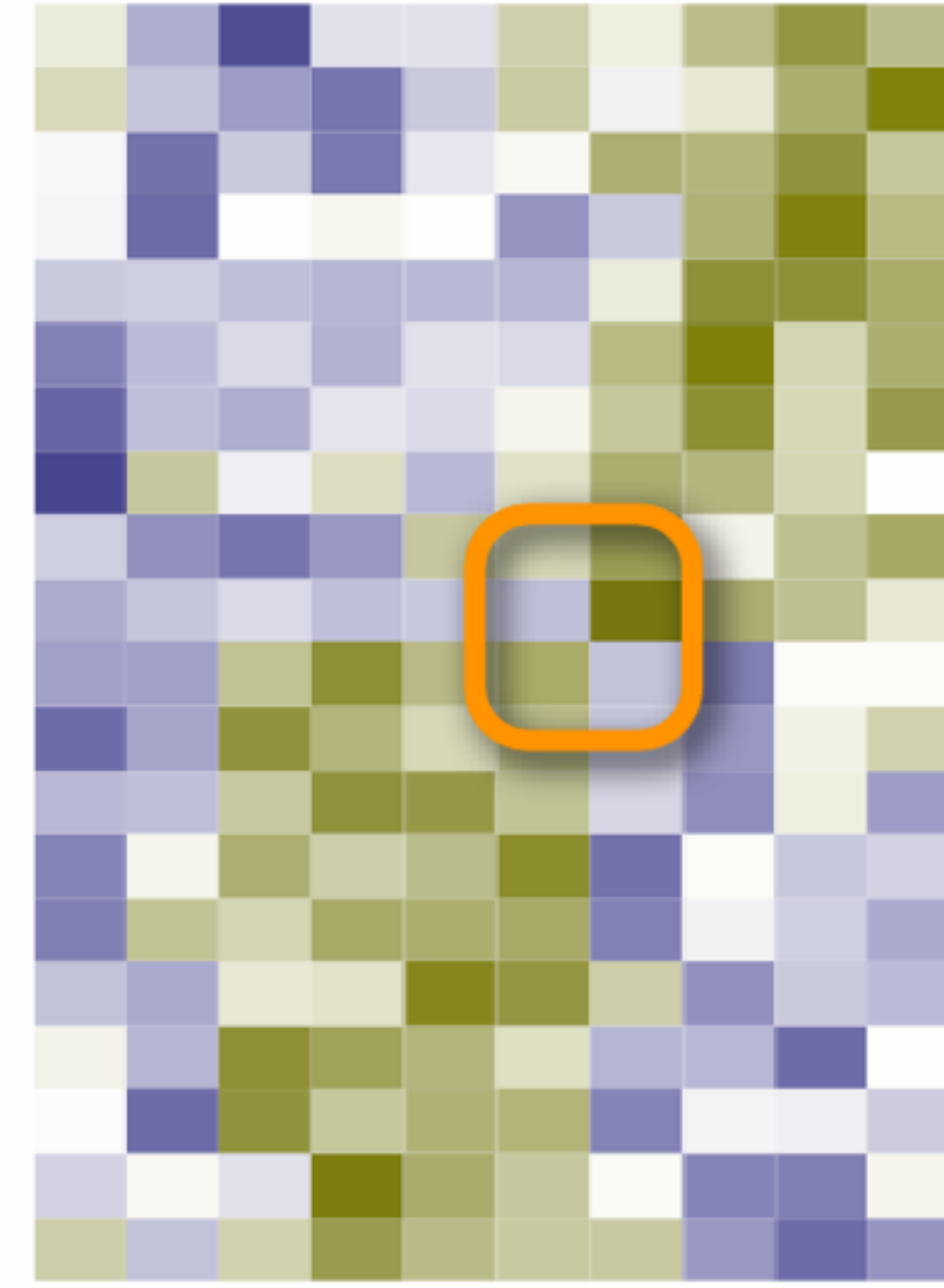


Deuteranope Vision  
("Red-Green Blindness")

# Good Color Mapping

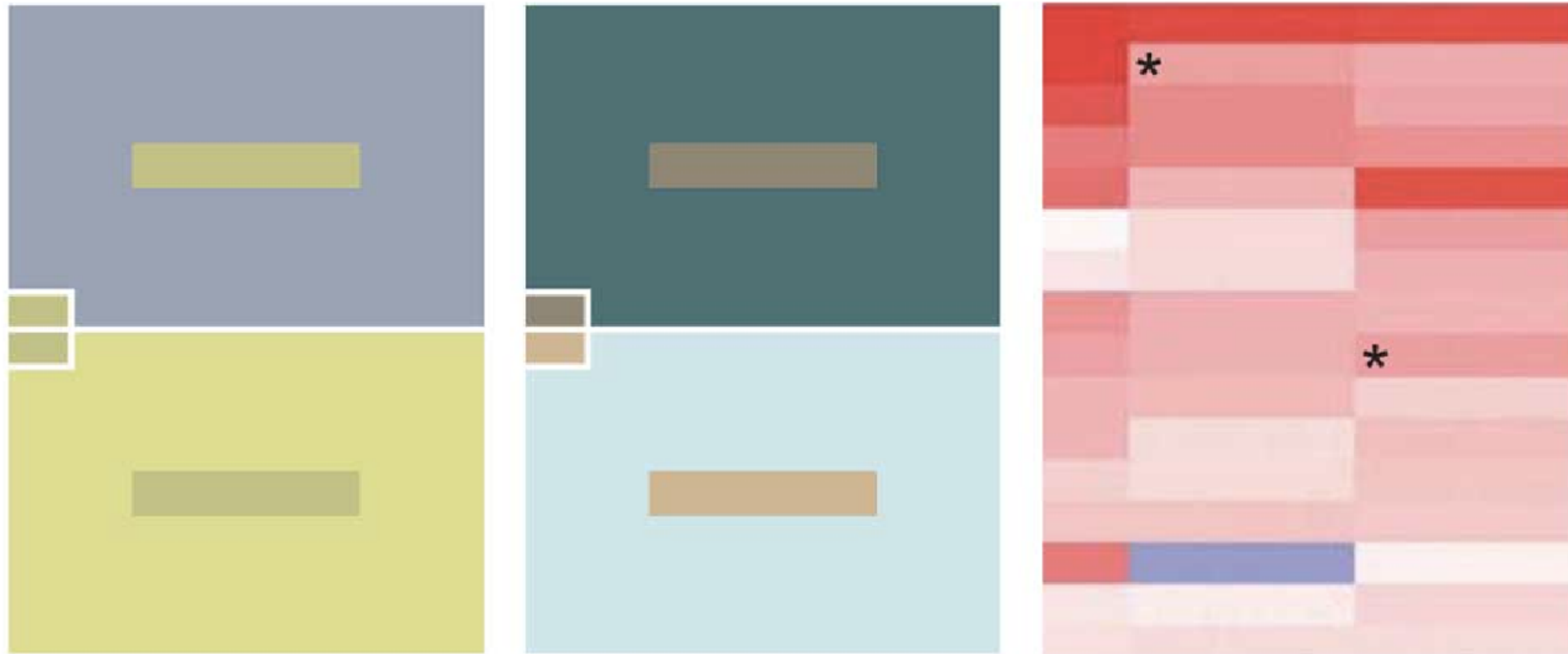


Normal Vision



Deuteranope Vision  
("Red-Green Blindness")

# Color is relative!





# Clustering

Classification of items into “similar” bins

Based on similarity measures

Euclidean distance, Pearson correlation, ...

Partitional Algorithms

divide data into set of bins

# bins either manually set (e.g., k-means) or automatically determined (e.g., affinity propagation)

Hierarchical Algorithms

Produce “similarity tree” – dendrogram

Bi-Clustering

Clusters dimensions & records

Fuzzy clustering

allows occurrence of elements in multiples clusters

# Clustering Applications

Clusters can be used to

- order (pixel based techniques)

- brush (geometric techniques)

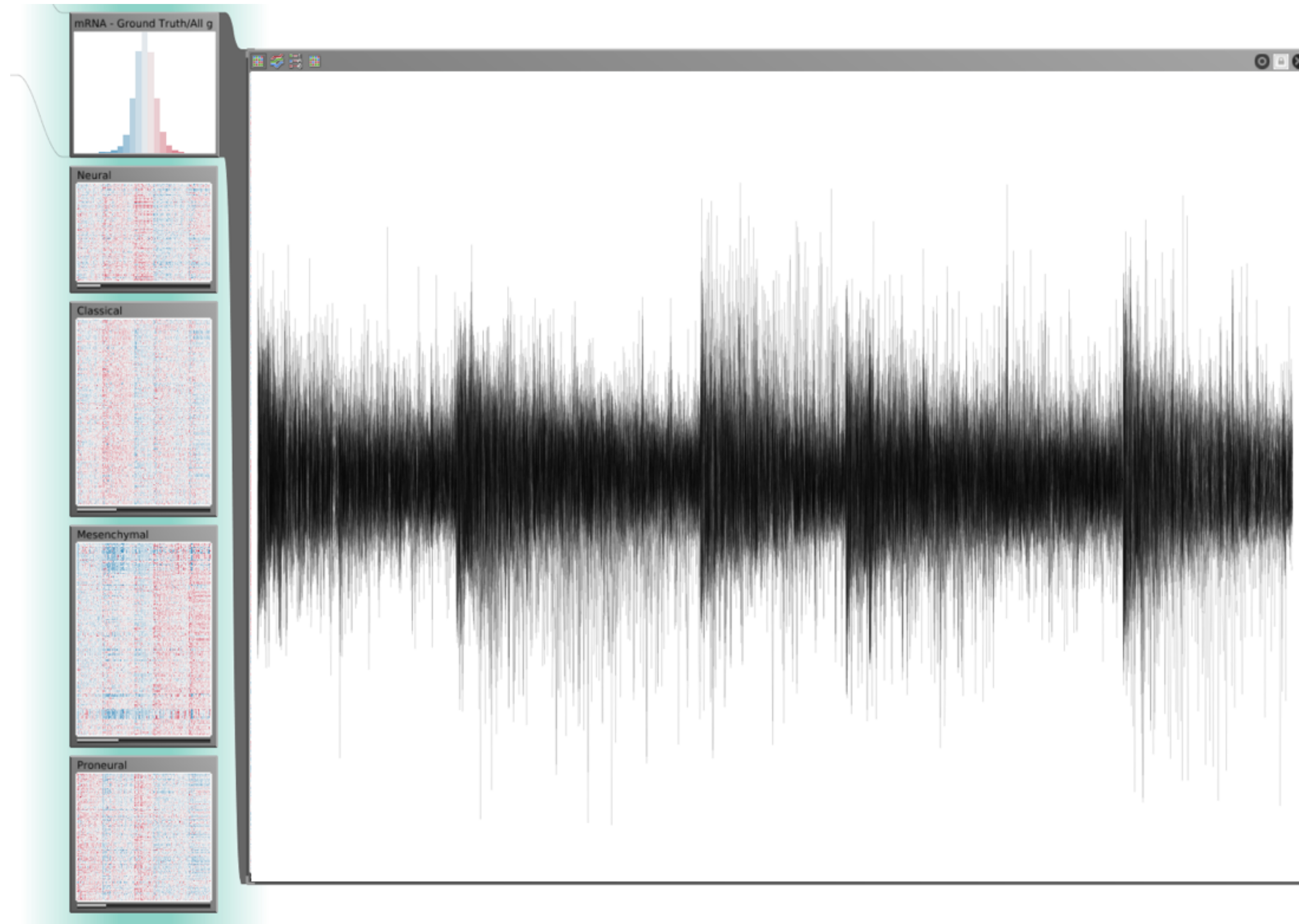
- aggregate

## Aggregation

- cluster more homogeneous than whole dataset

- statistical measures, distributions, etc. more meaningful

# Clustered Heat Map



# Dimensionality Reduction



# Dimensionality Reduction

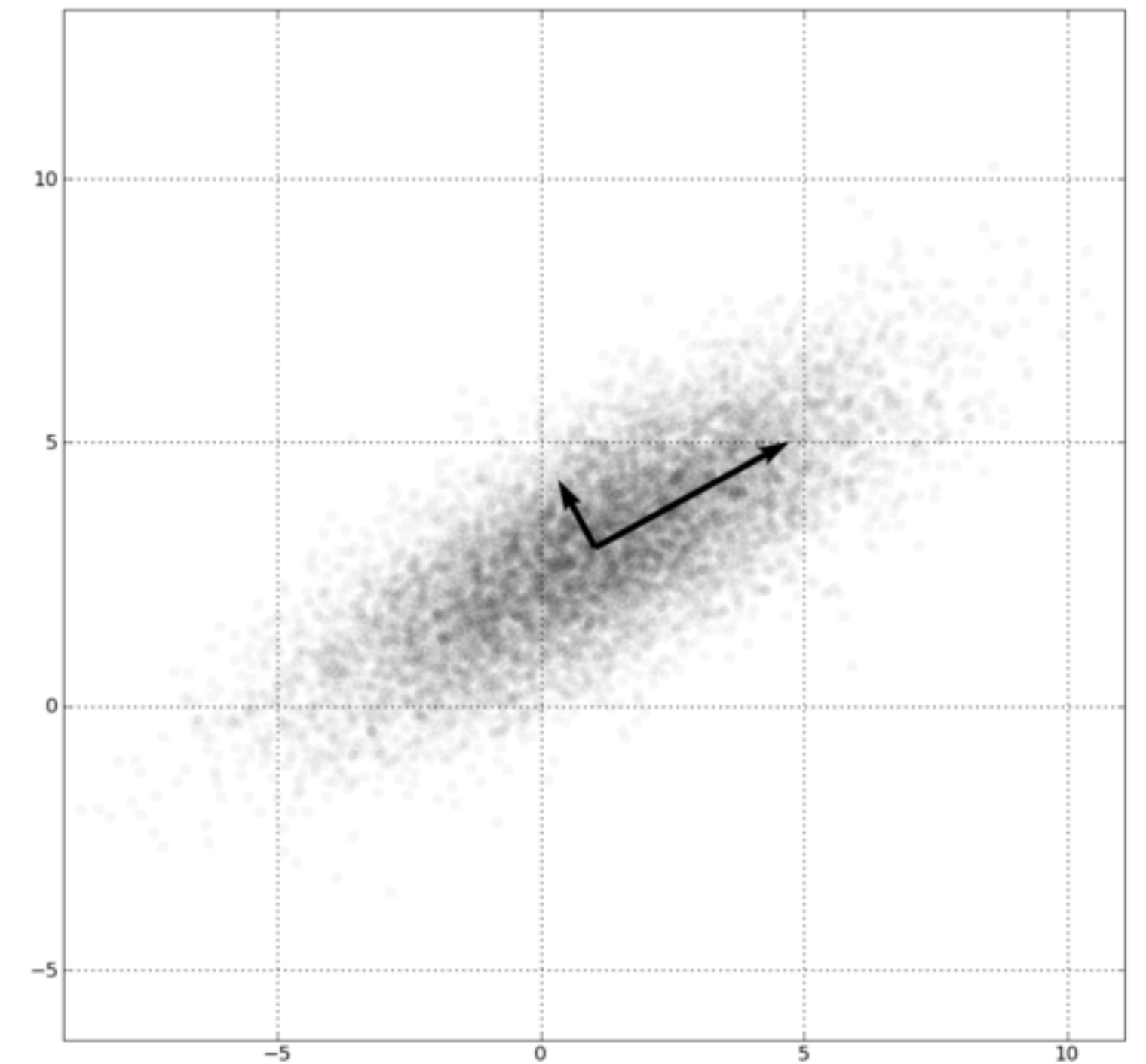
Reduce high dimensional to lower dimensional space

Preserve as much of variation as possible

Plot lower dimensional space

*Principal Component Analysis (PCA)*

linear mapping, by order of variance



# Multidimensional Scaling

Nonlinear, better suited for  
some DS

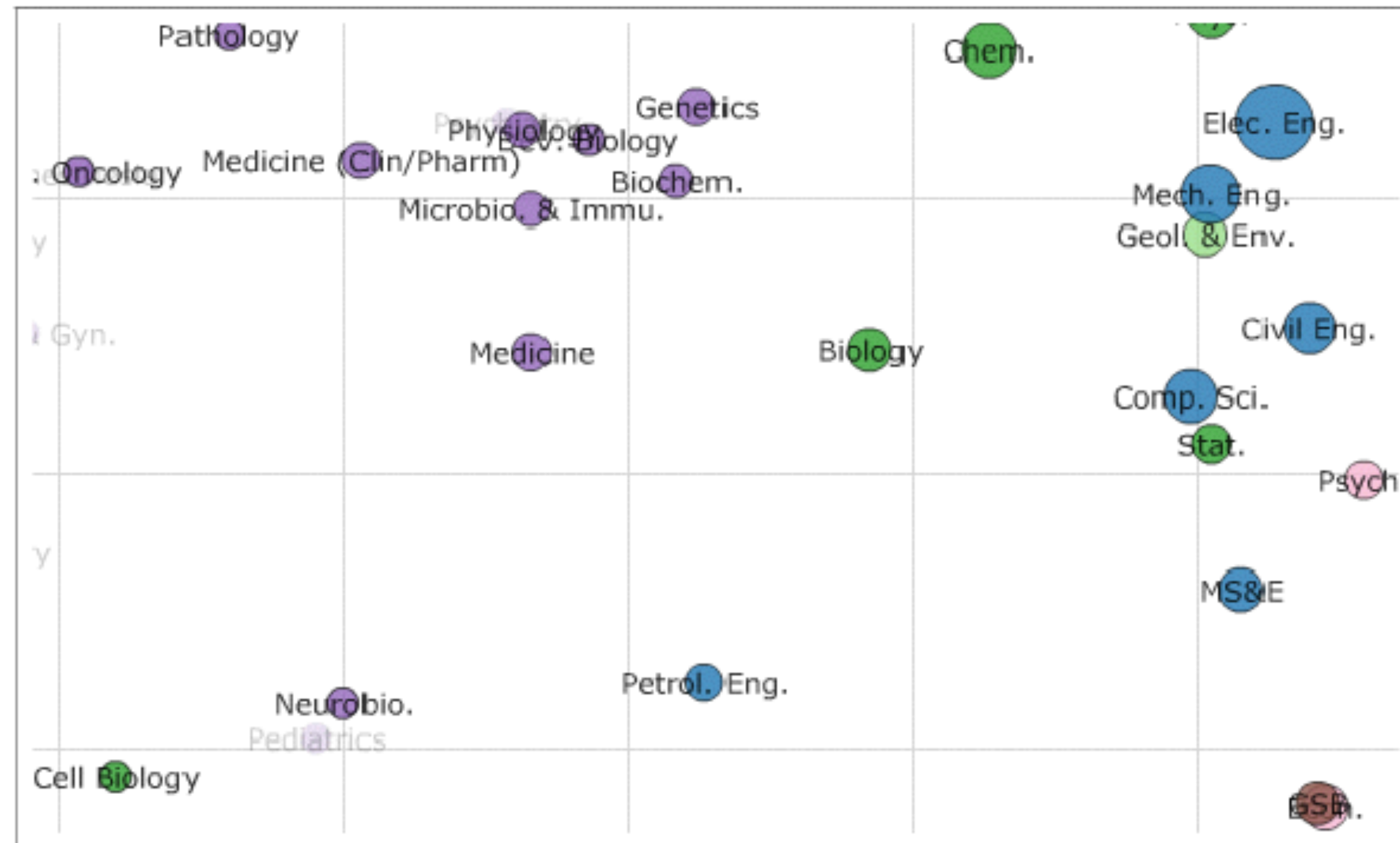
Popular for text analysis



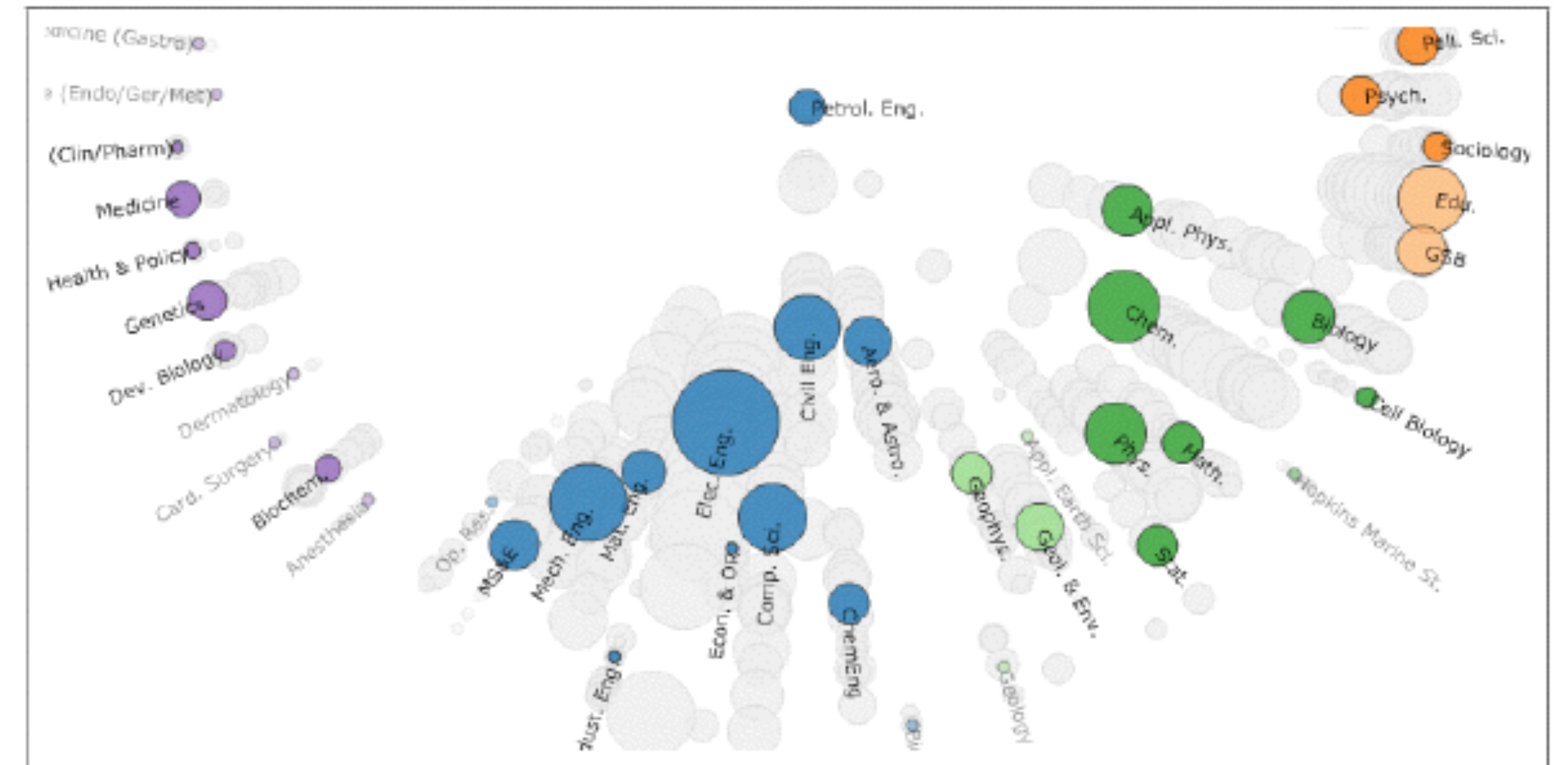
[Doerk 2011]

# Can we Trust Dimensionality Reduction?

Topical distances between departments in a 2D projection



Topical distances between the selected Petroleum Engineering and the others.

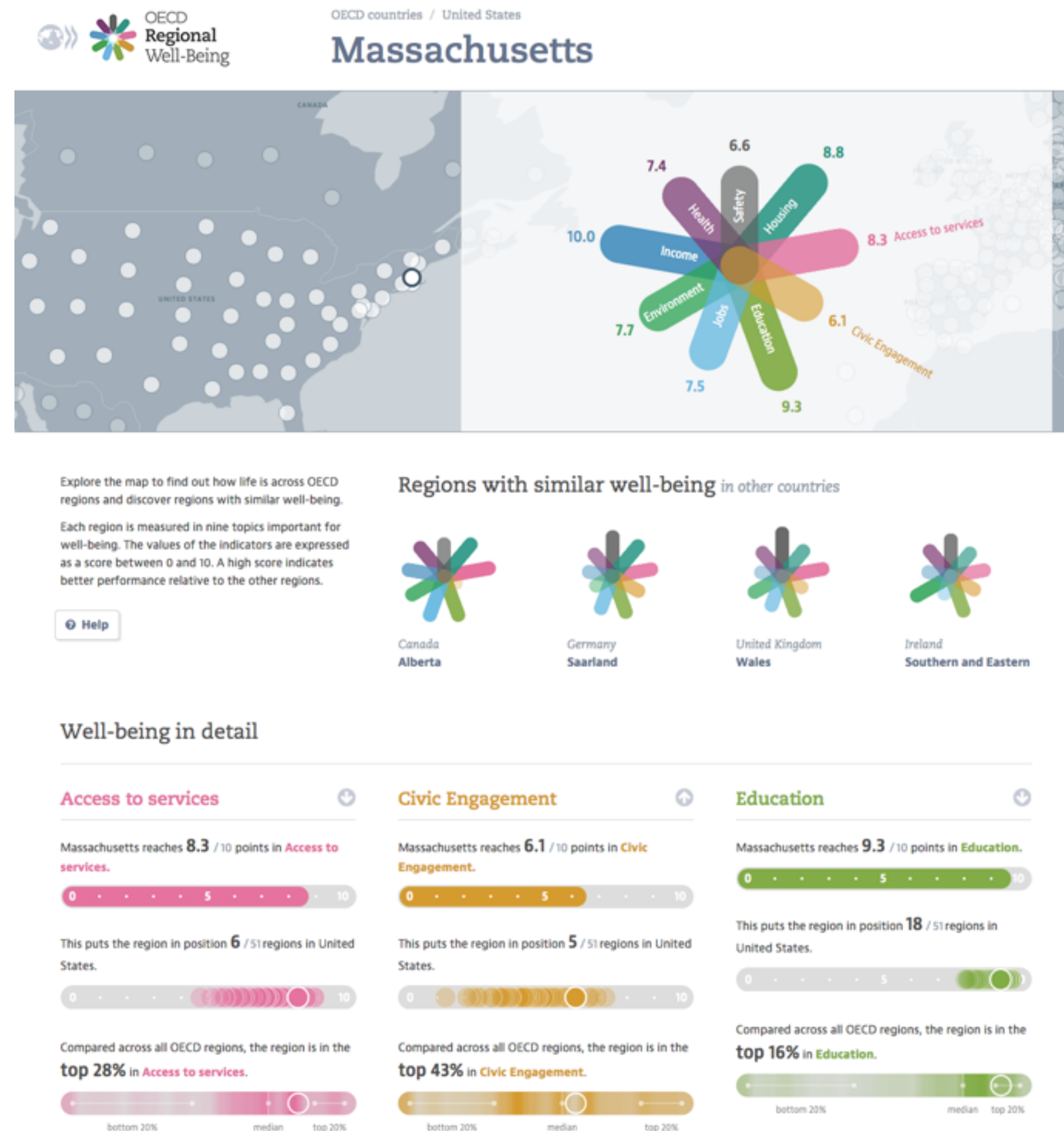


[Chuang et al., 2012]

# Design Critique



# OECD: <http://goo.gl/QfxHfv>



<http://www.oecdregionalwellbeing.org/>

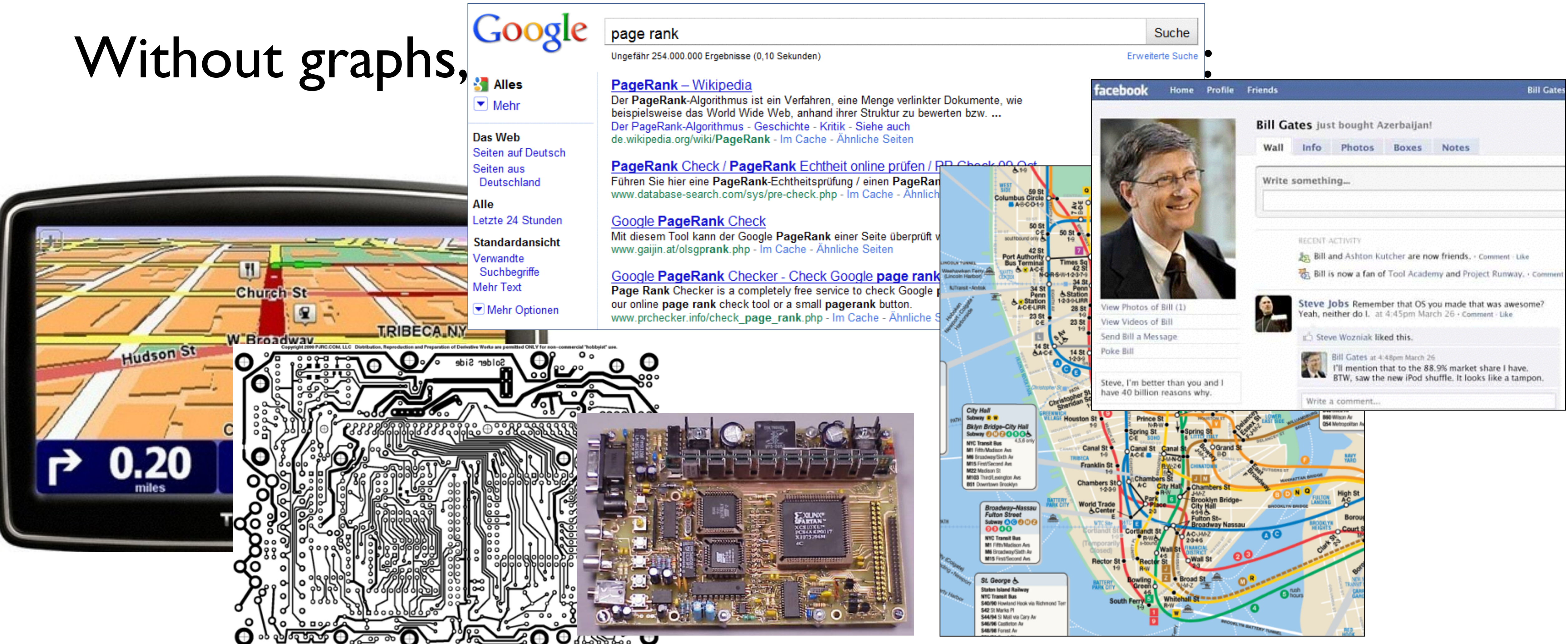
# Graph Visualization

Based on Slides by HJ Schulz and M Streit

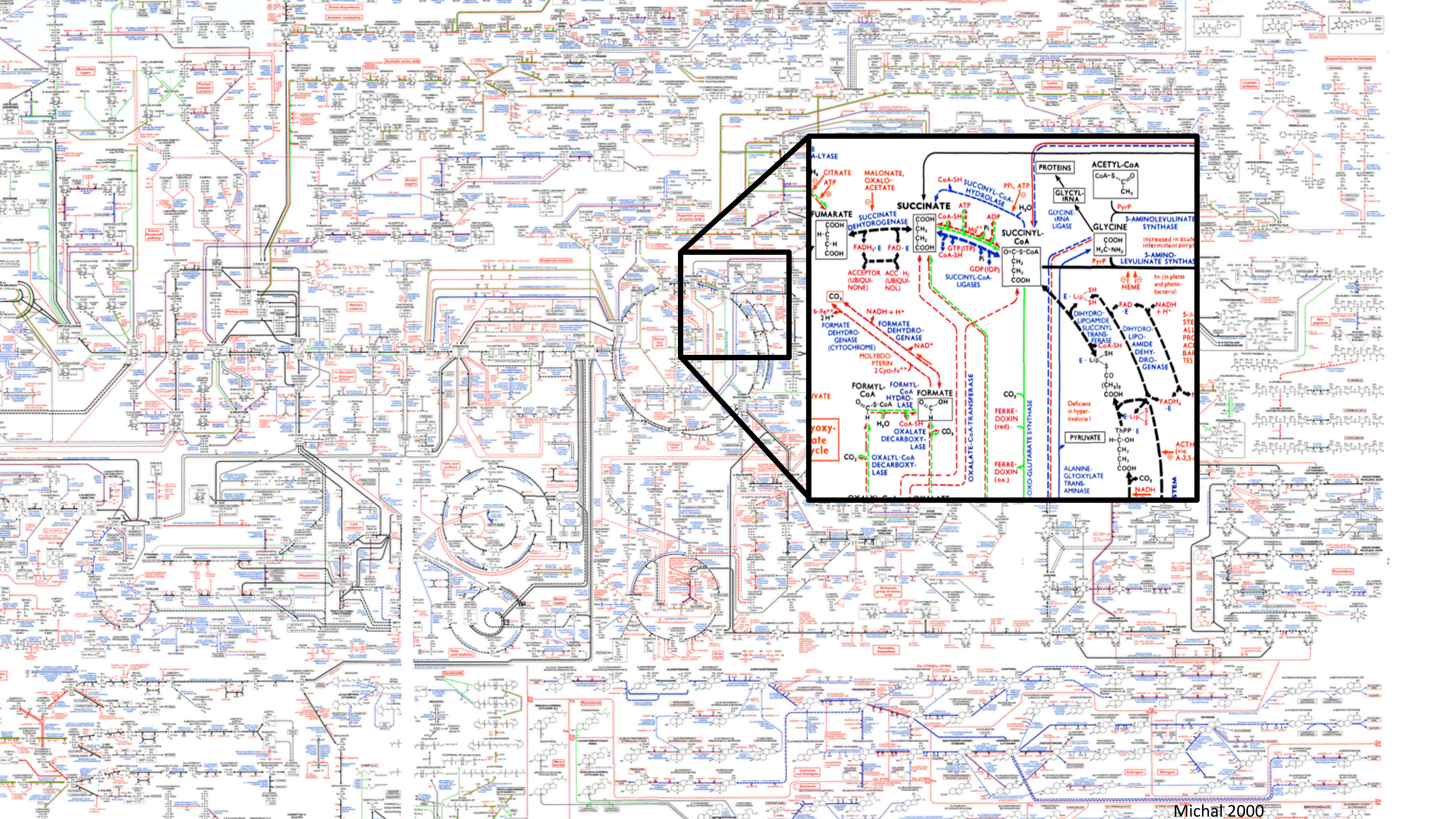


# Applications of Graphs

Without graphs,





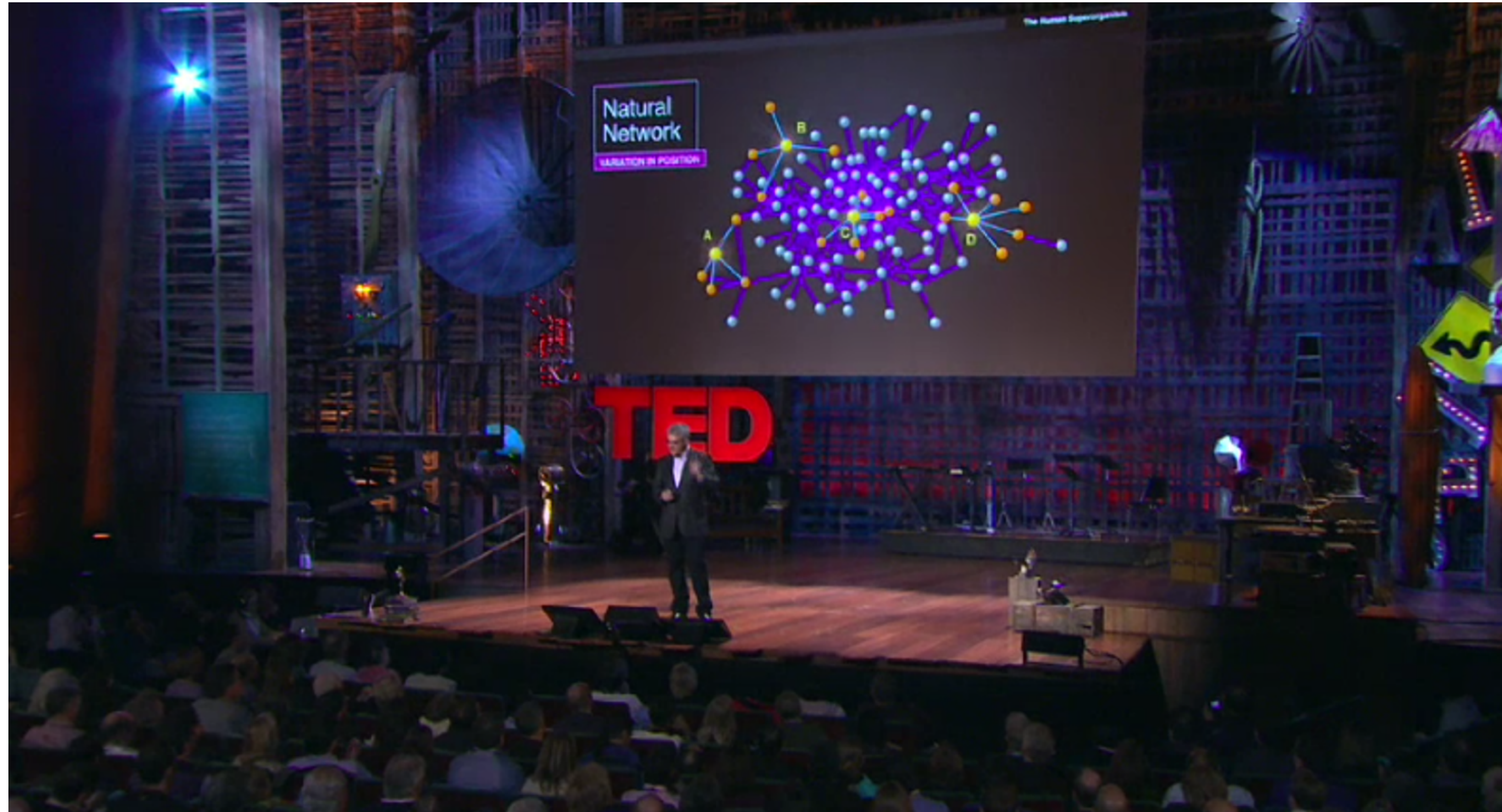








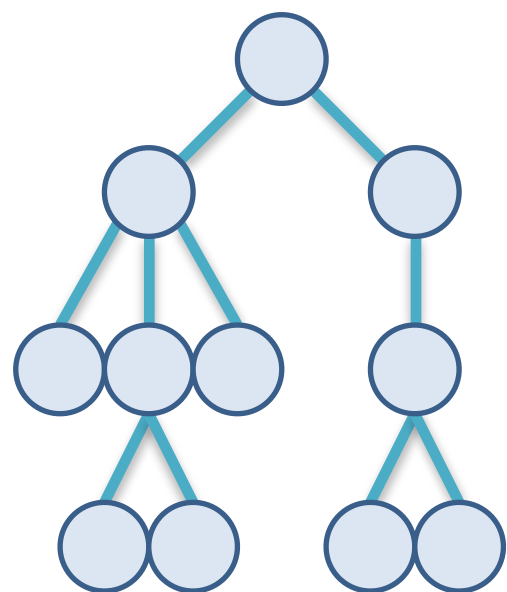
# Graph Visualization Case Study



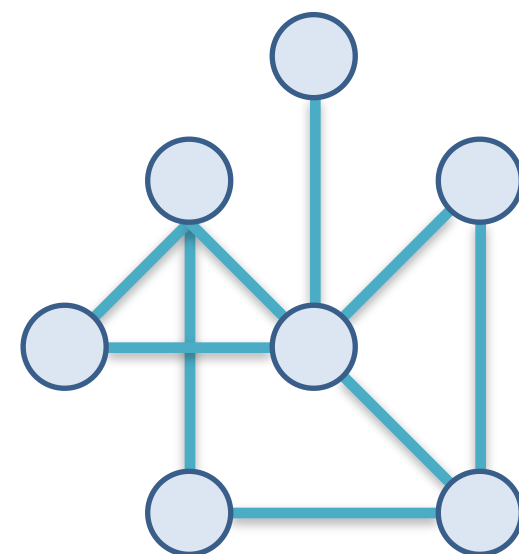


# Graph Theory Fundamentals

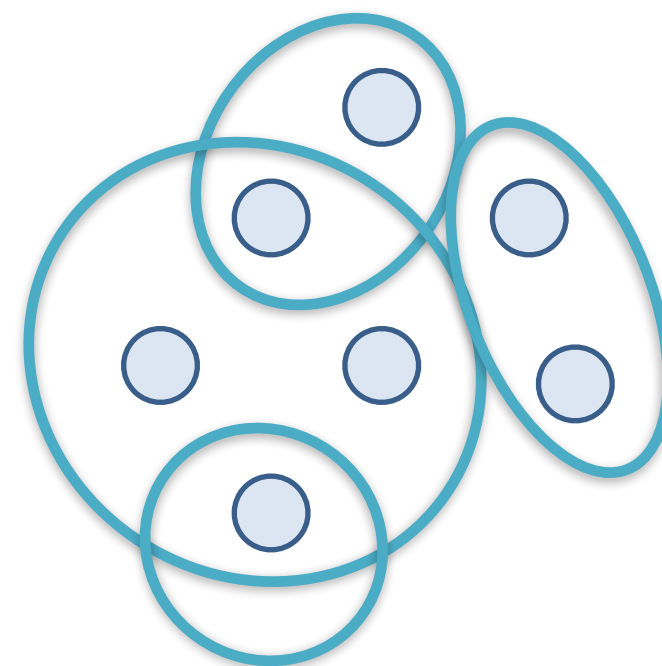
Tree



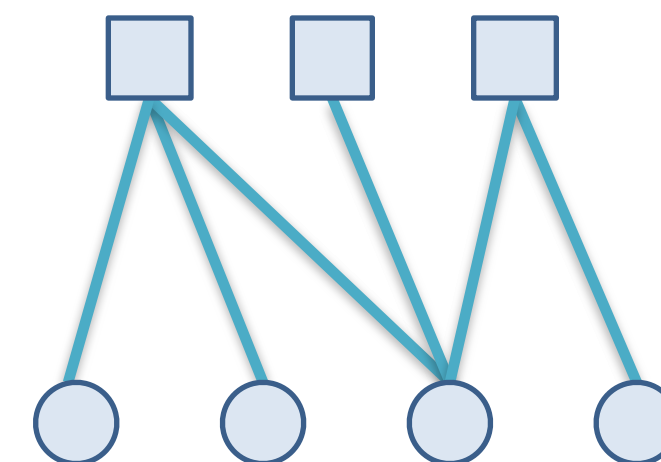
Network



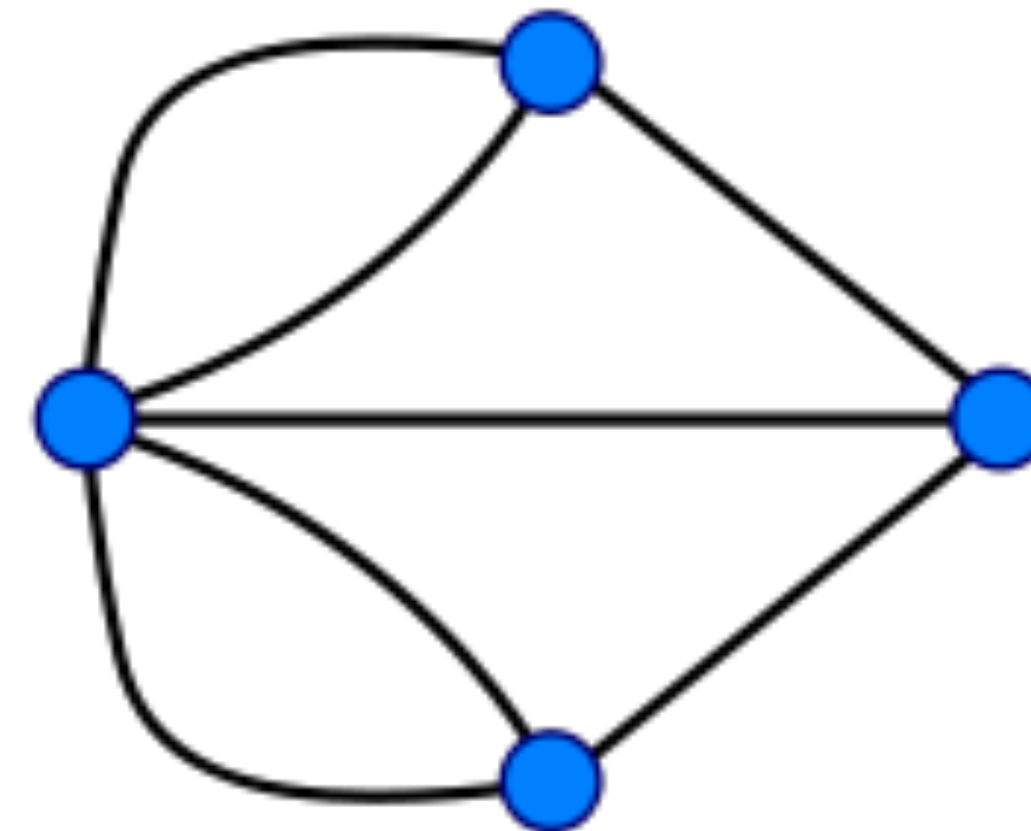
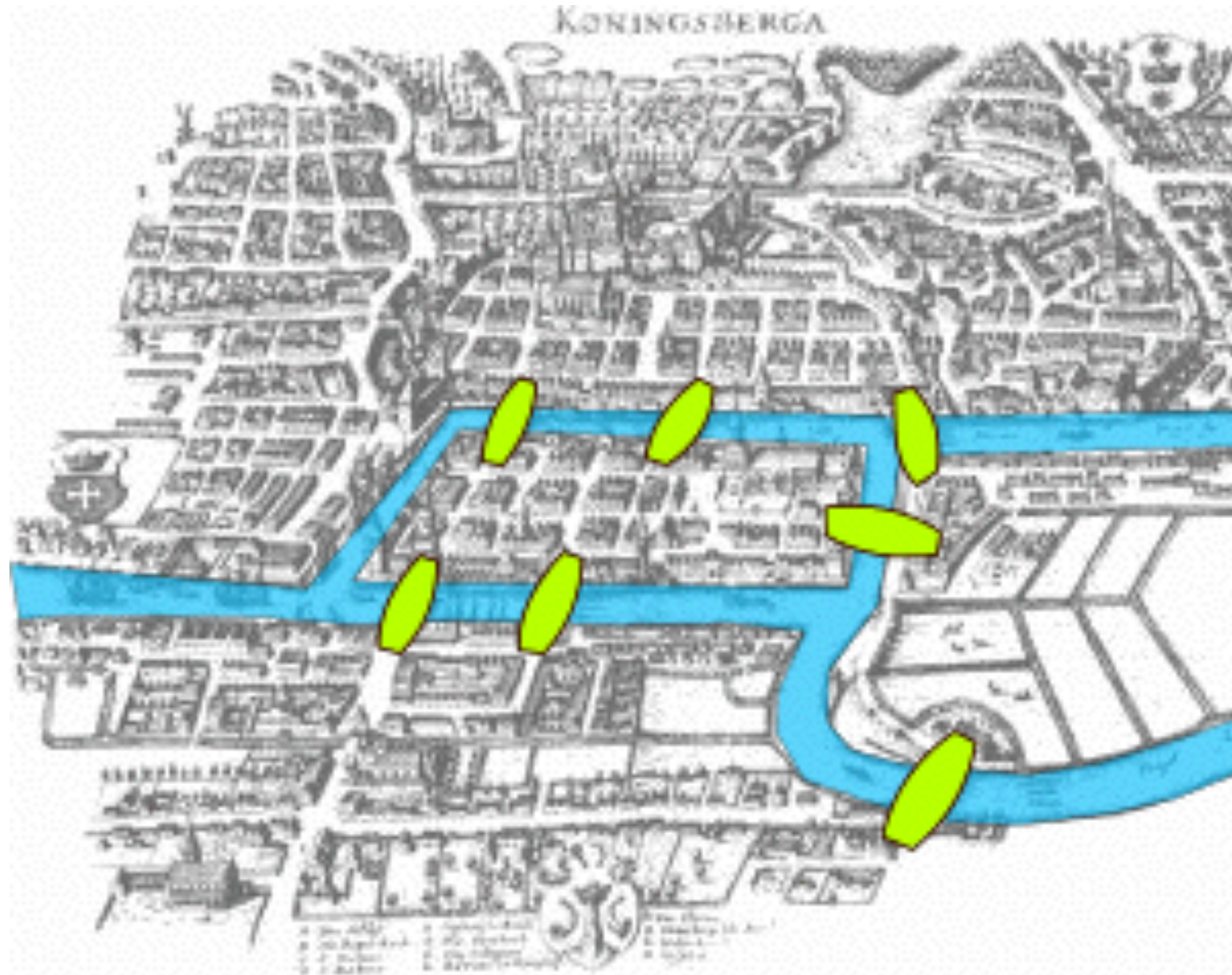
Hypergraph



Bipartite Graph



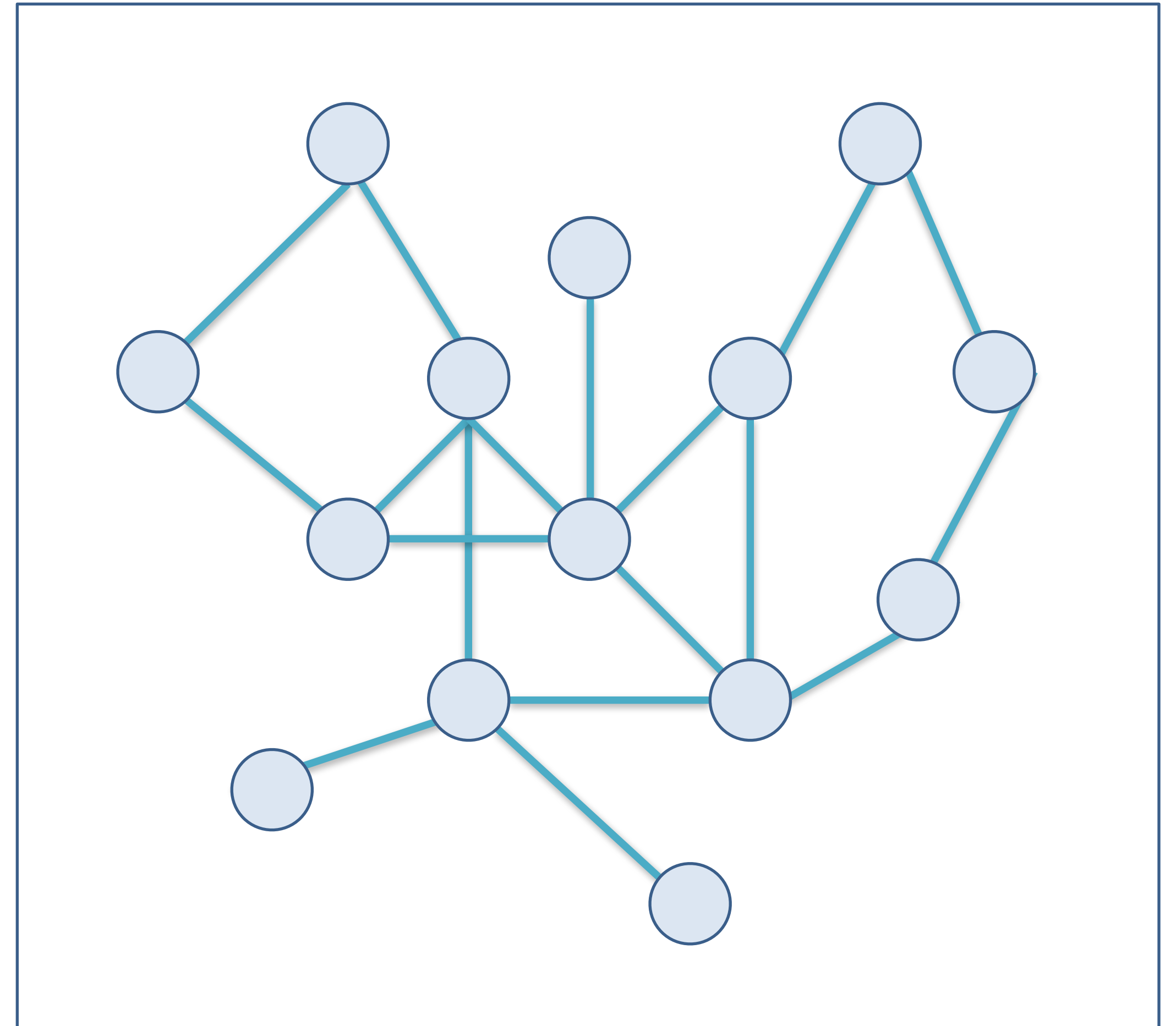
# Königsberg Bridge Problem (1736)



Find a Hamiltonian Path (path that visits each vertex exactly once).  
Want to make 1 million \$? Develop  $O(n^k)$  algorithm.

# Graph Terms (1)

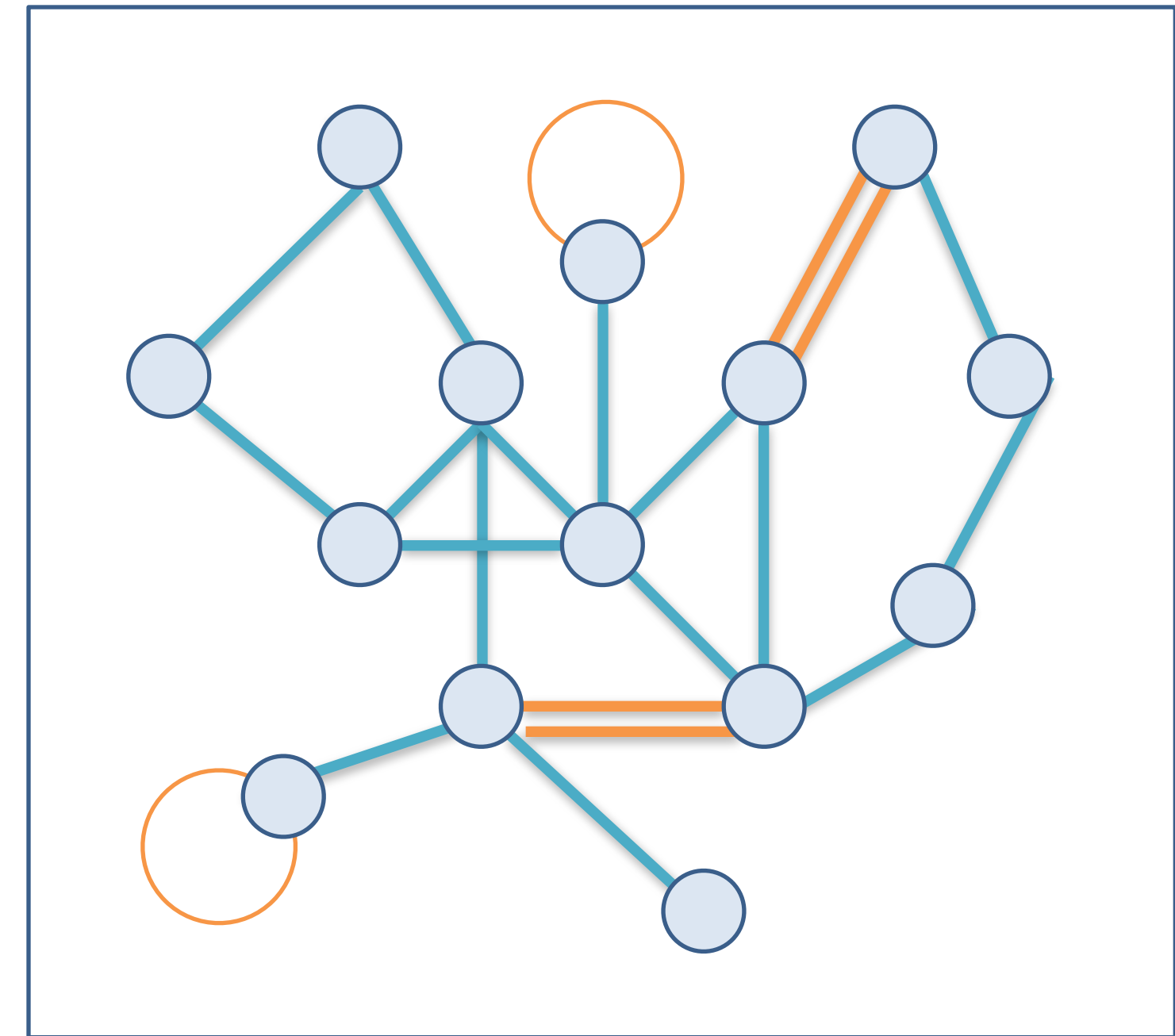
A graph  $G(V,E)$  consists of a set of **vertices**  $V$  (also called nodes) and a set of **edges**  $E$  connecting these vertices.





# Graph Terms (2)

A simple graph  $G(V,E)$  is a graph which contains **no multi-edges** and **no loops**

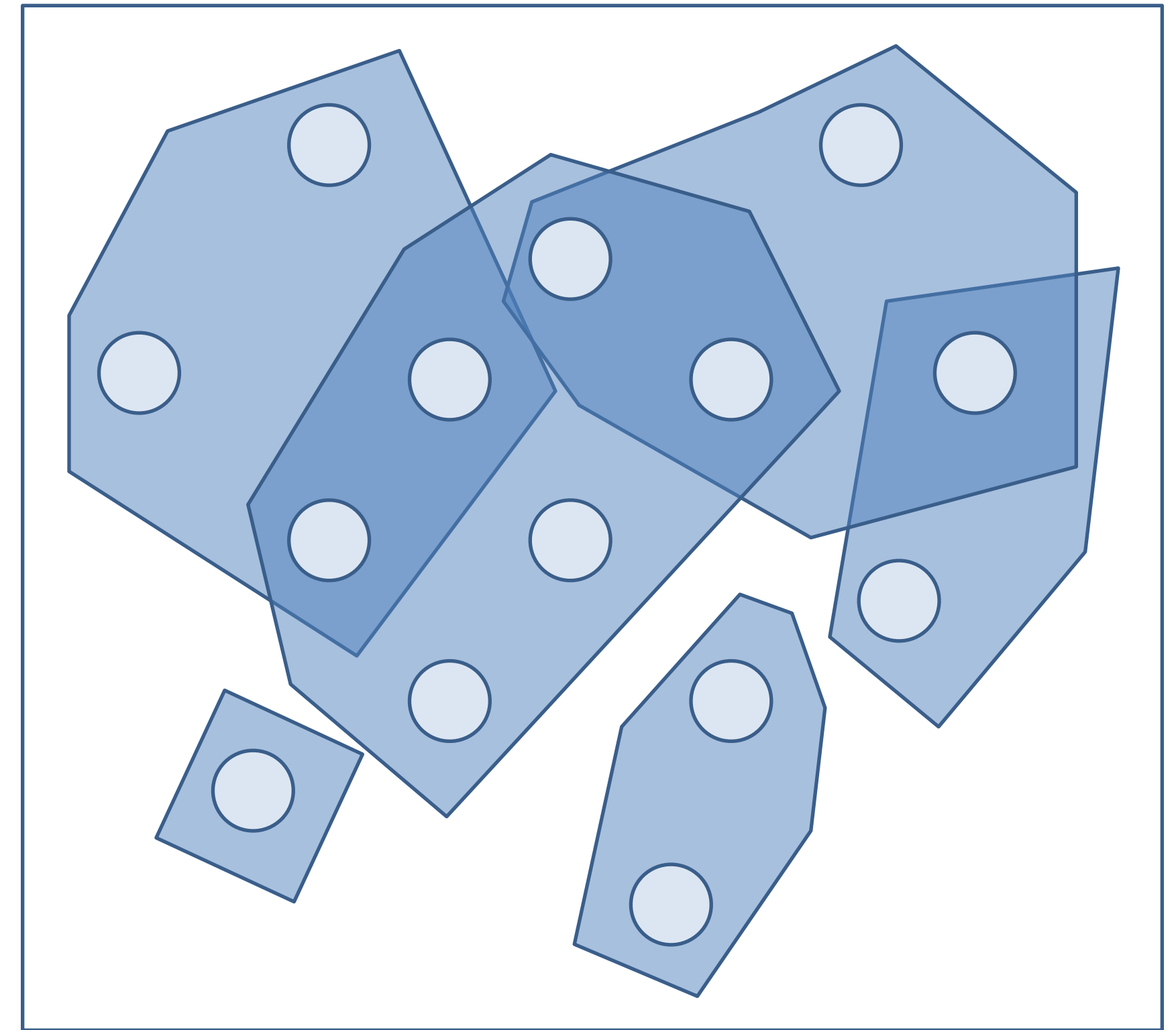


Not a simple graph!  
→ A ***general graph***

# Graph Terms (3)

A directed graph (digraph) is a graph that discerns between the edges  $A \rightarrow B$  and  $A \leftarrow B$ .

A hypergraph is a graph with edges connecting any number of vertices.

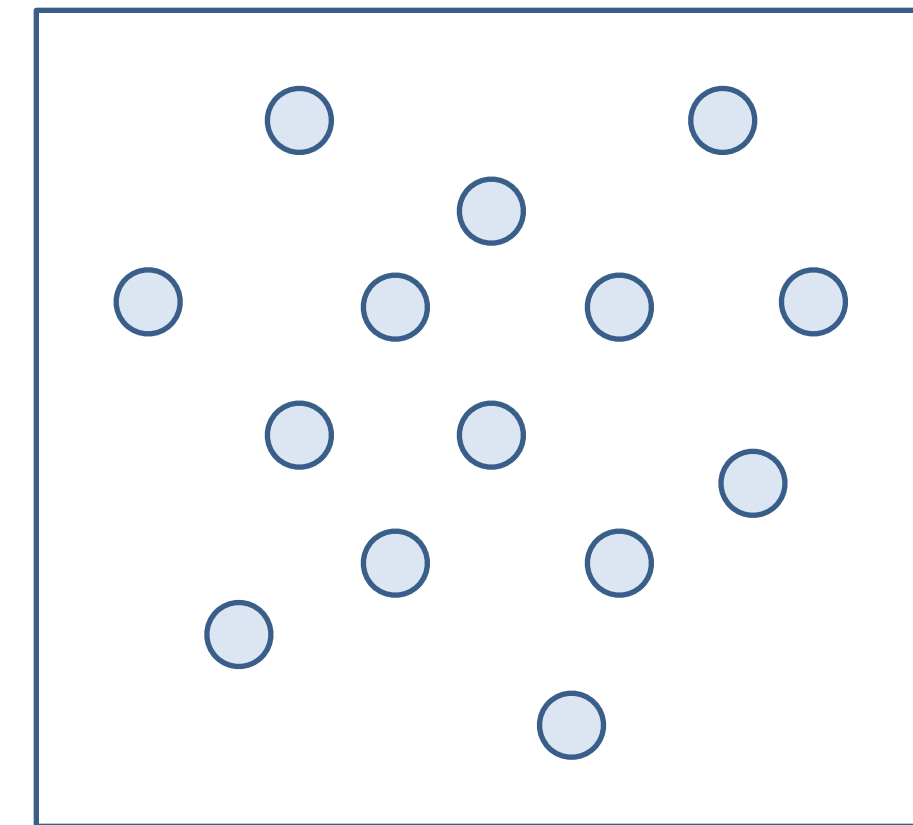


Hypergraph Example

# Graph Terms (4)

## ***Independent Set***

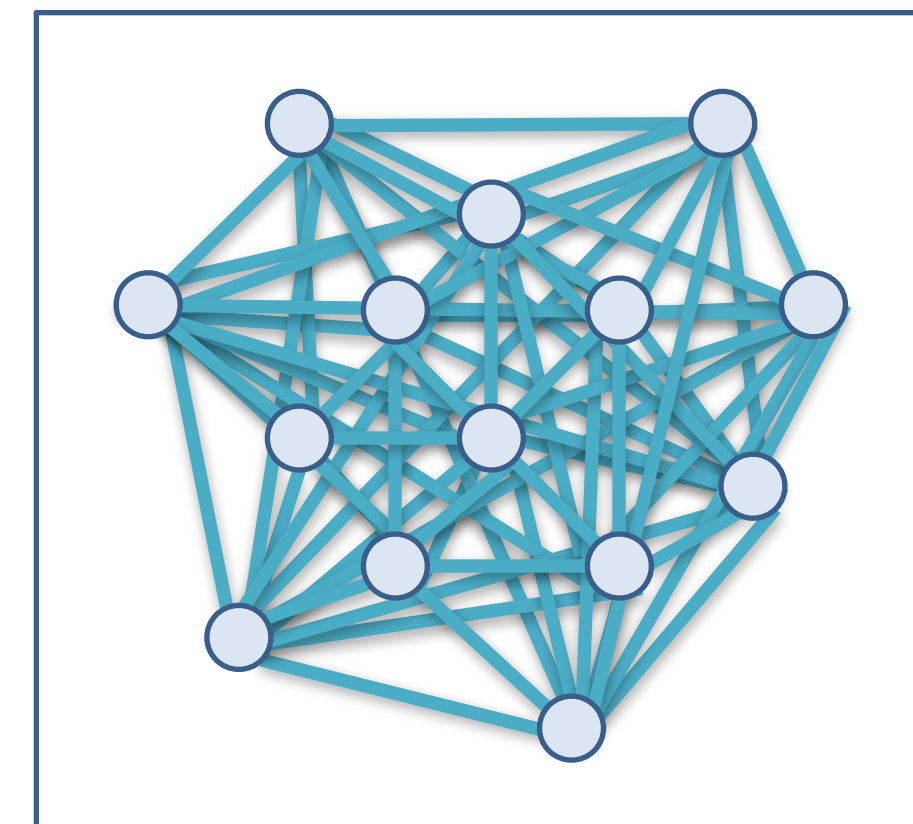
G contains no edges



Independent Set

## ***Clique***

G contains all possible edges



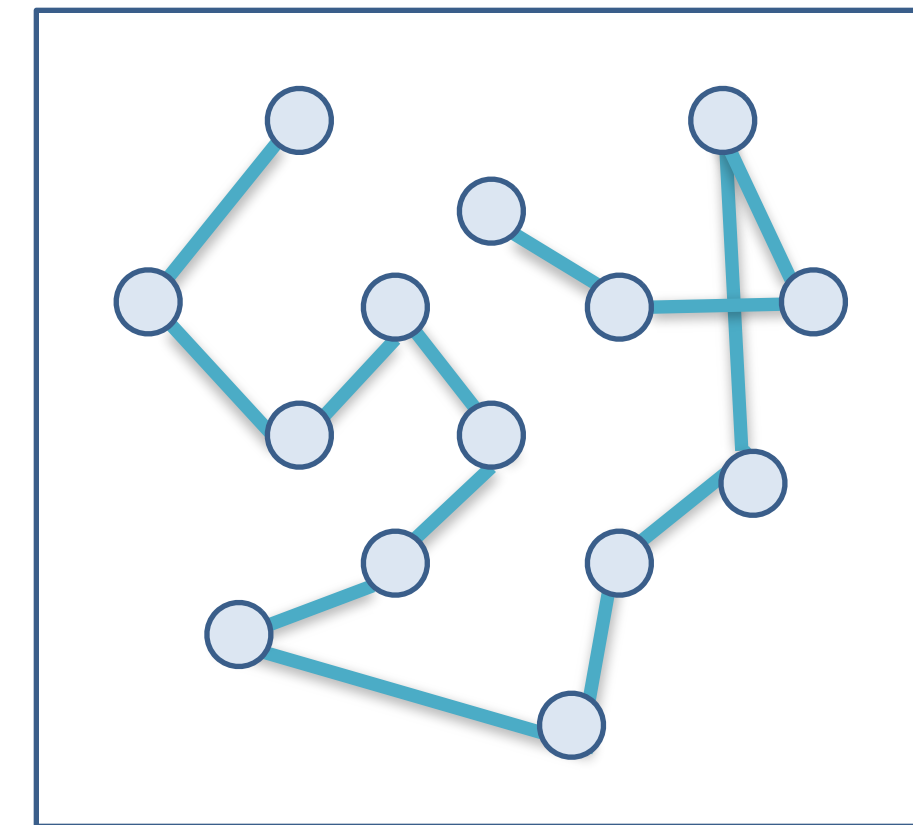
Clique



# Graph Terms (5)

## ***Path***

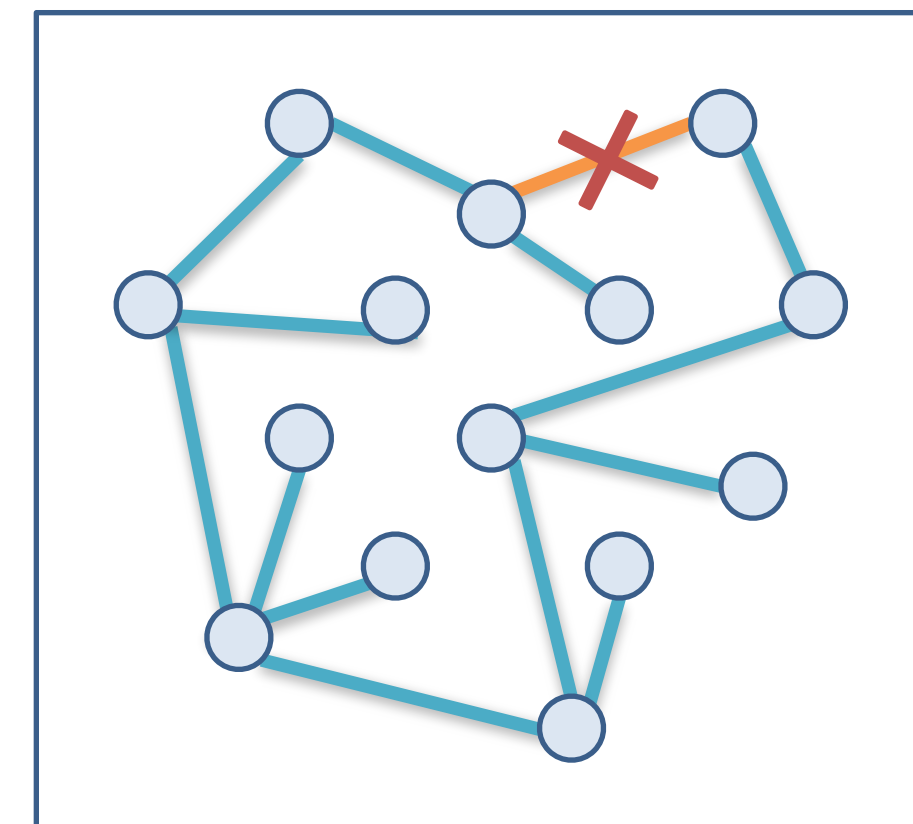
G contains only edges that can be consecutively traversed



Path

## ***Tree***

G contains no cycles



Tree

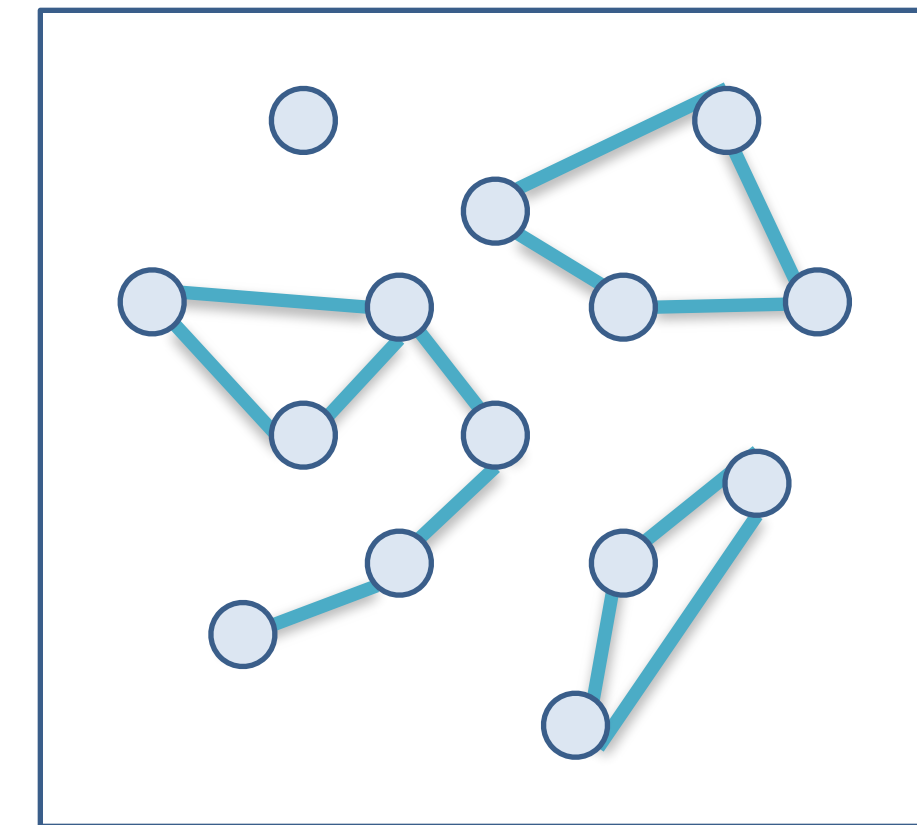
## ***Network***

G contains cycles

# Graph Terms (6)

## ***Unconnected graph***

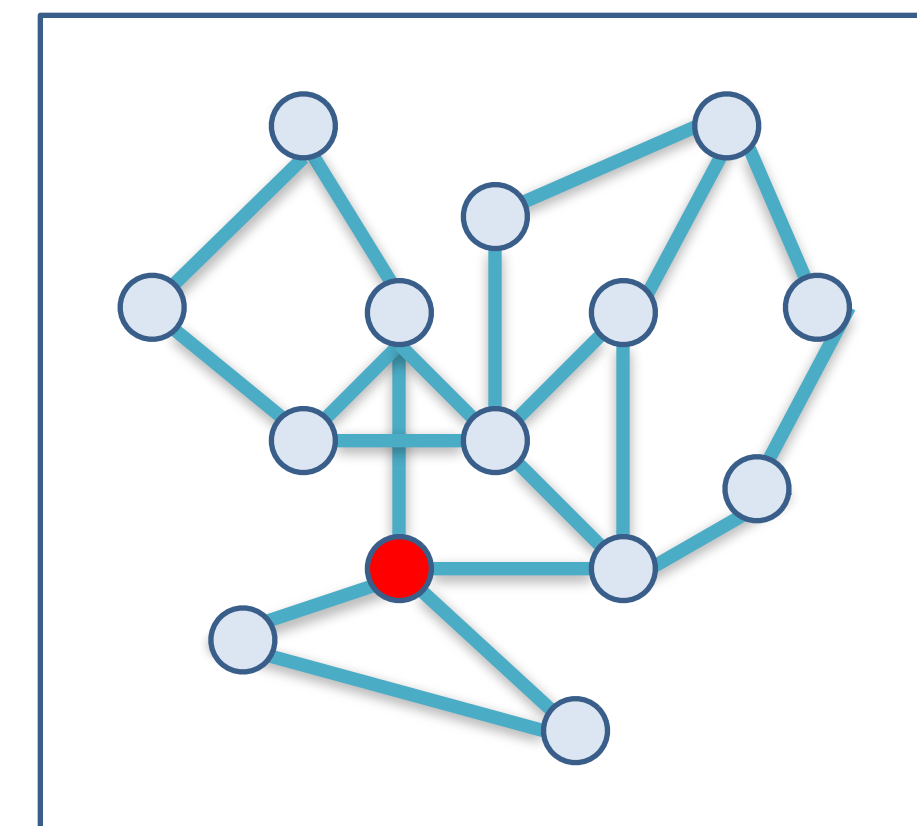
An edge traversal starting from a given vertex cannot reach any other vertex.



Unconnected Graph

## ***Articulation point***

Vertices, which if deleted from the graph, would break up the graph in multiple sub-graphs.

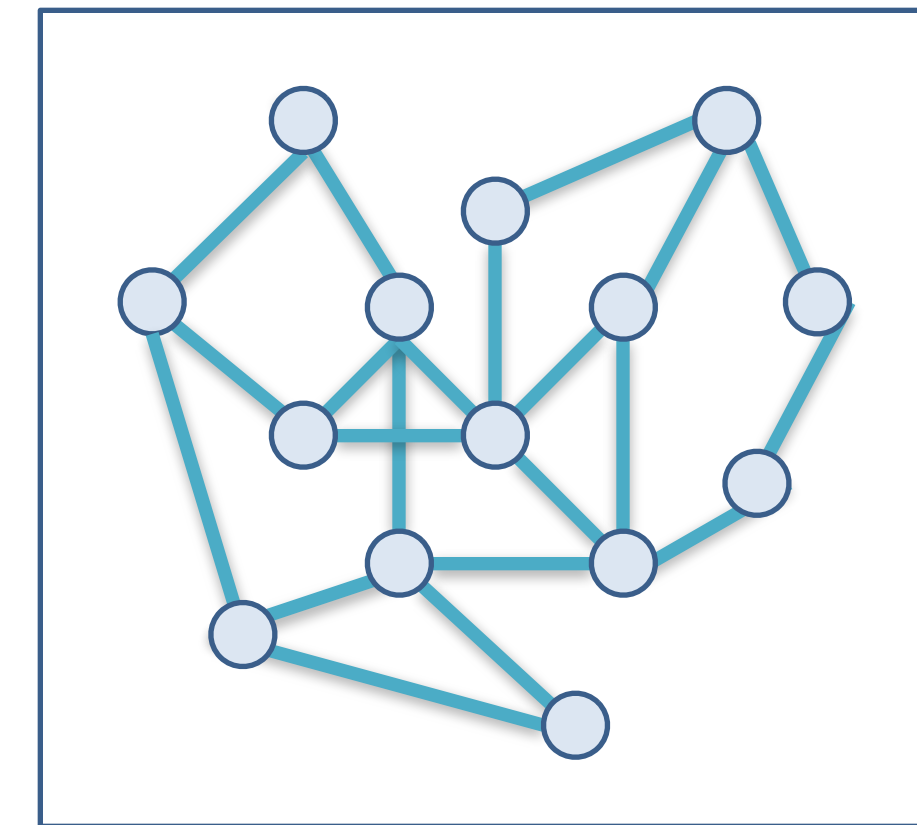


Articulation Point (red)

# Graph Terms (7)

## ***Biconnected graph***

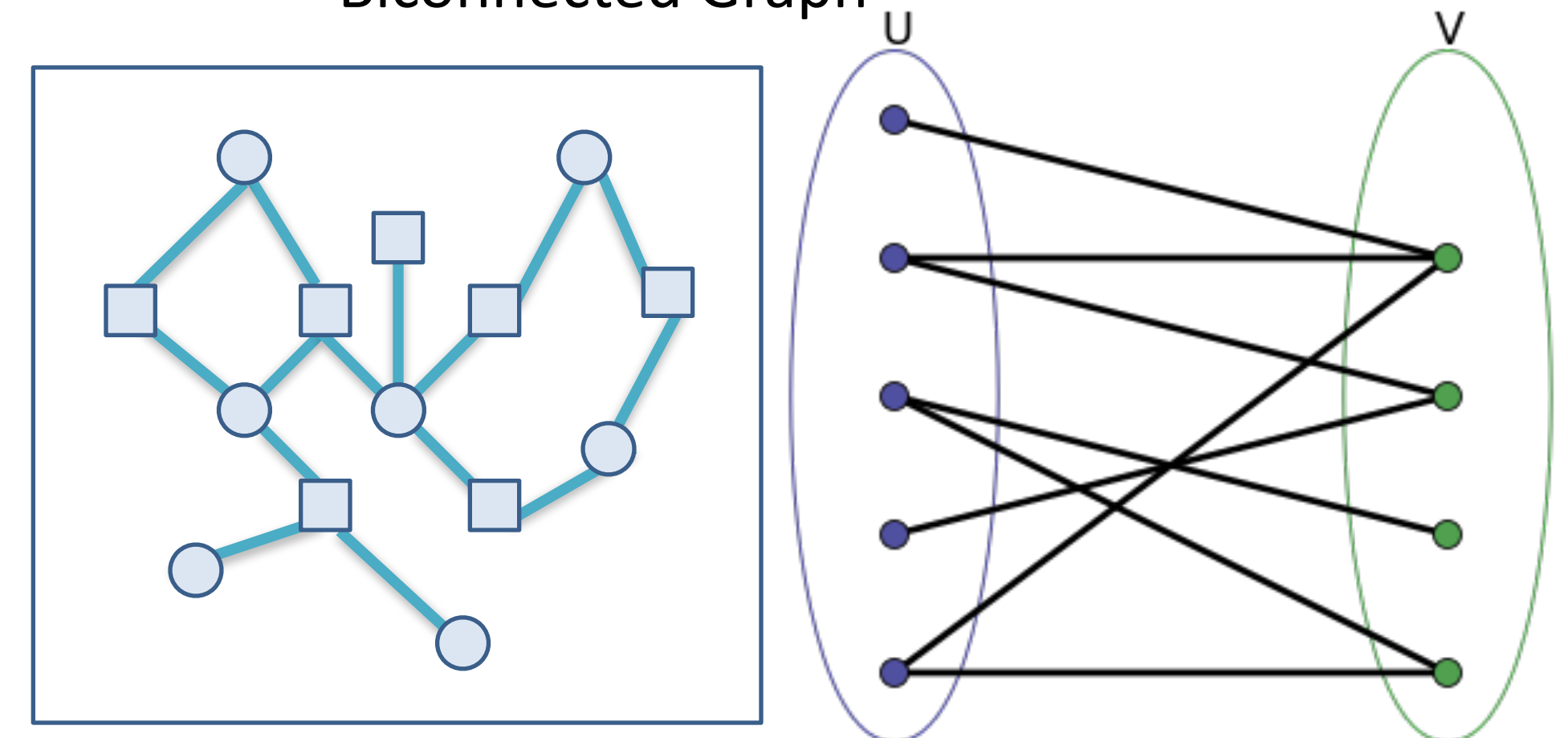
A graph without articulation points.



Biconnected Graph

## ***Bipartite graph***

The vertices can be partitioned in two independent sets.



Bipartite Graph



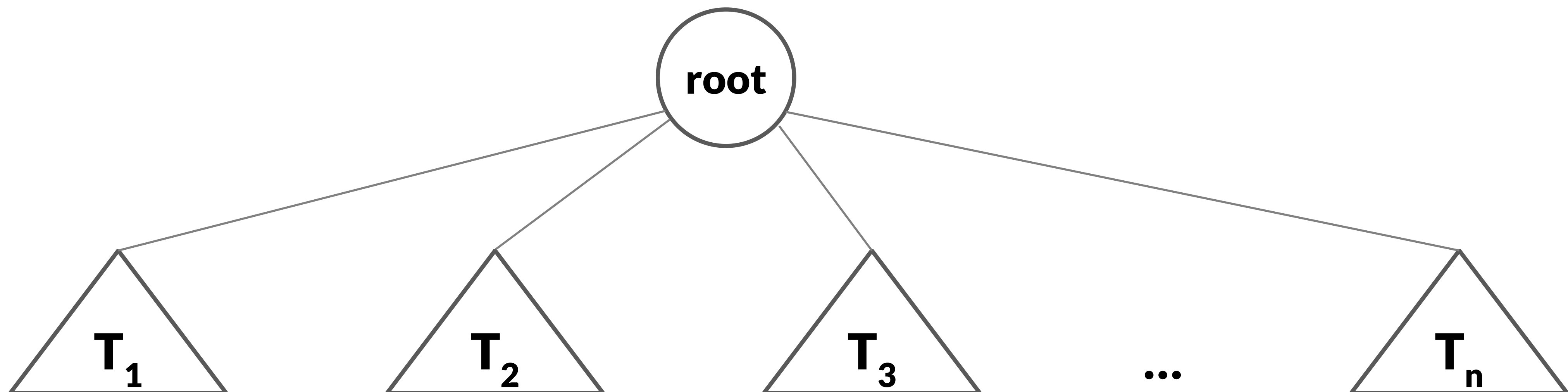
# Tree

**A graph with no cycles - or:**

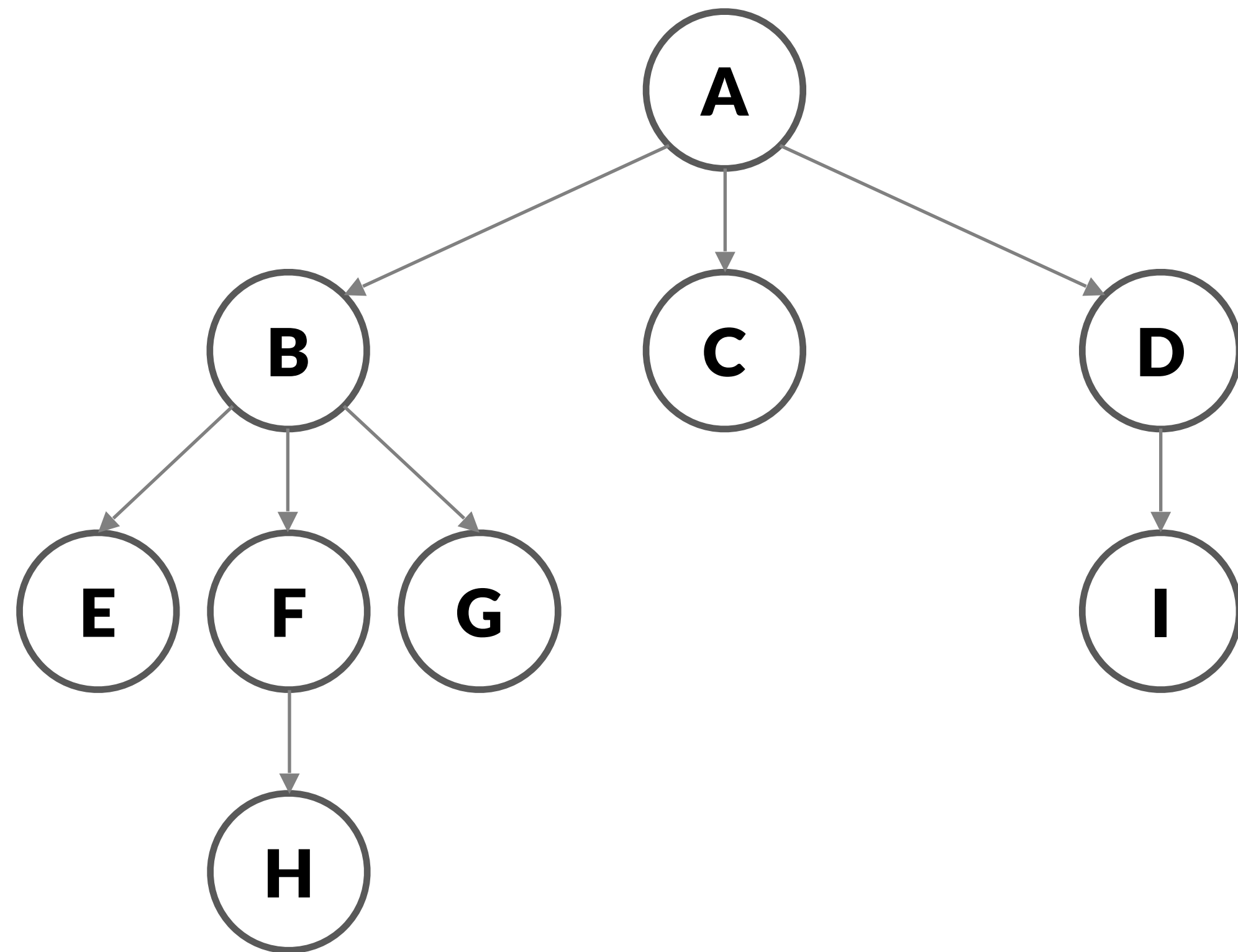
**A collection of nodes**

**contains a root node and 0-n subtrees**

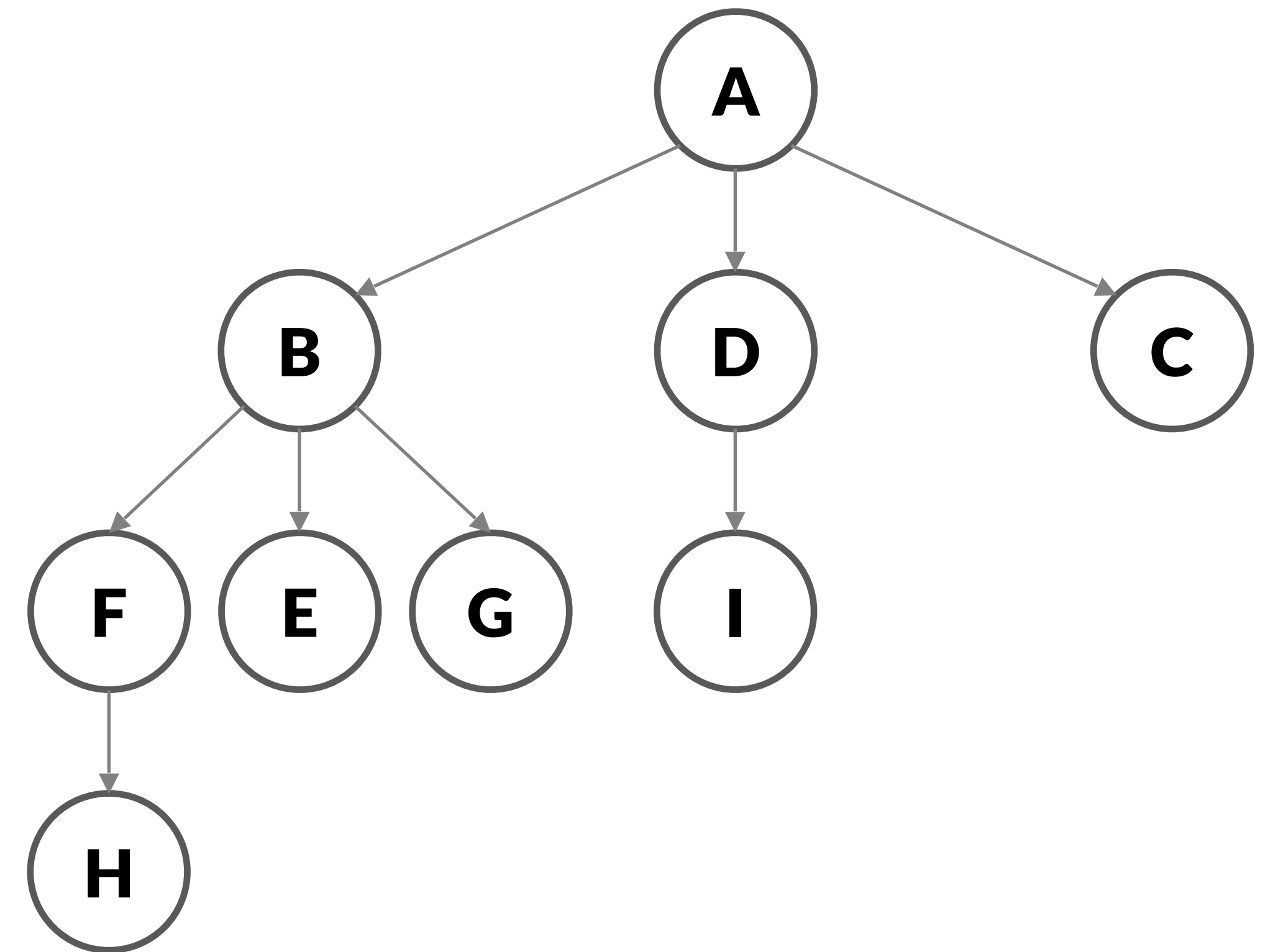
**subtrees are connected to root by an edge**



# Ordered Tree



≠



# Binary Trees

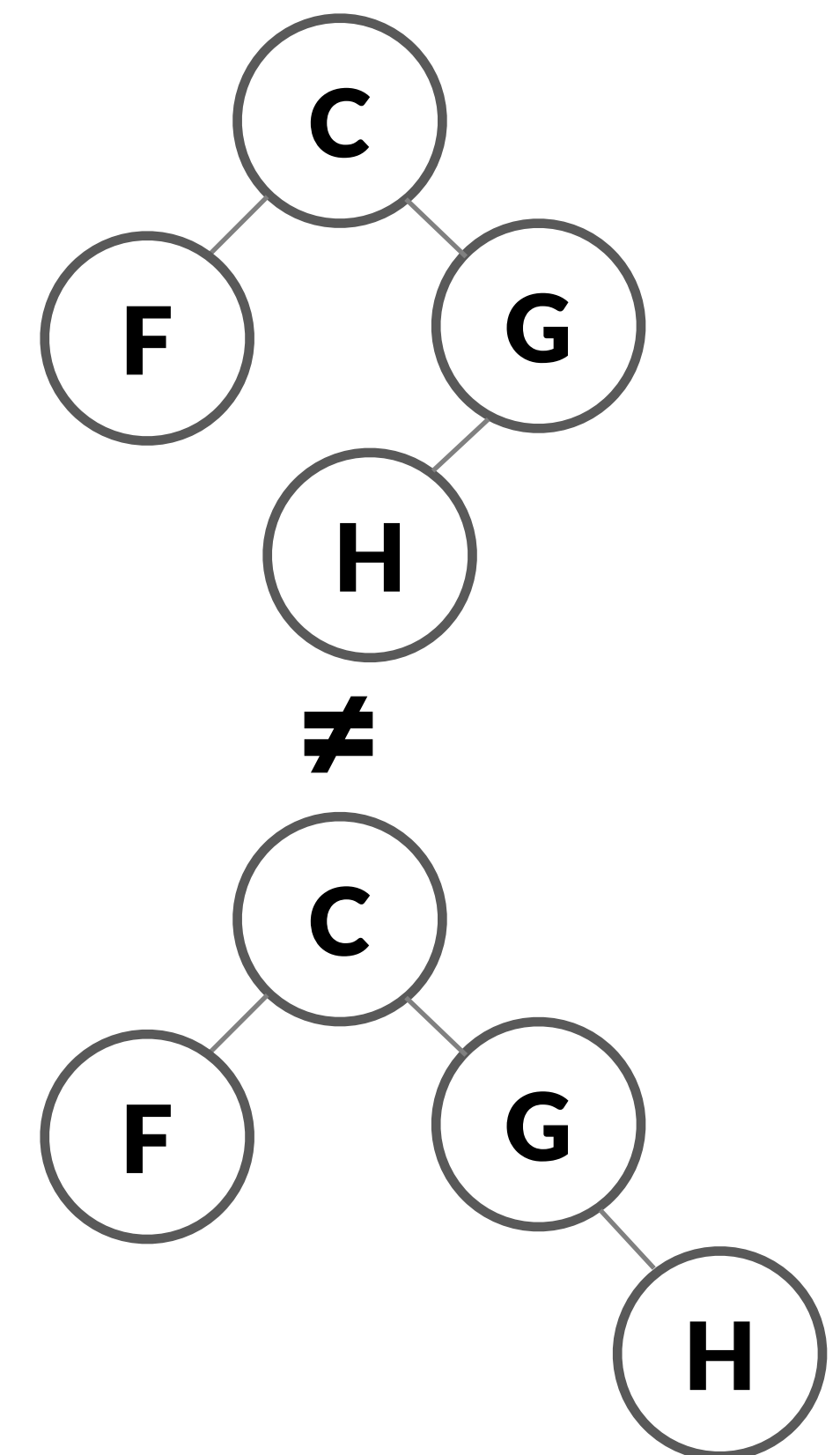
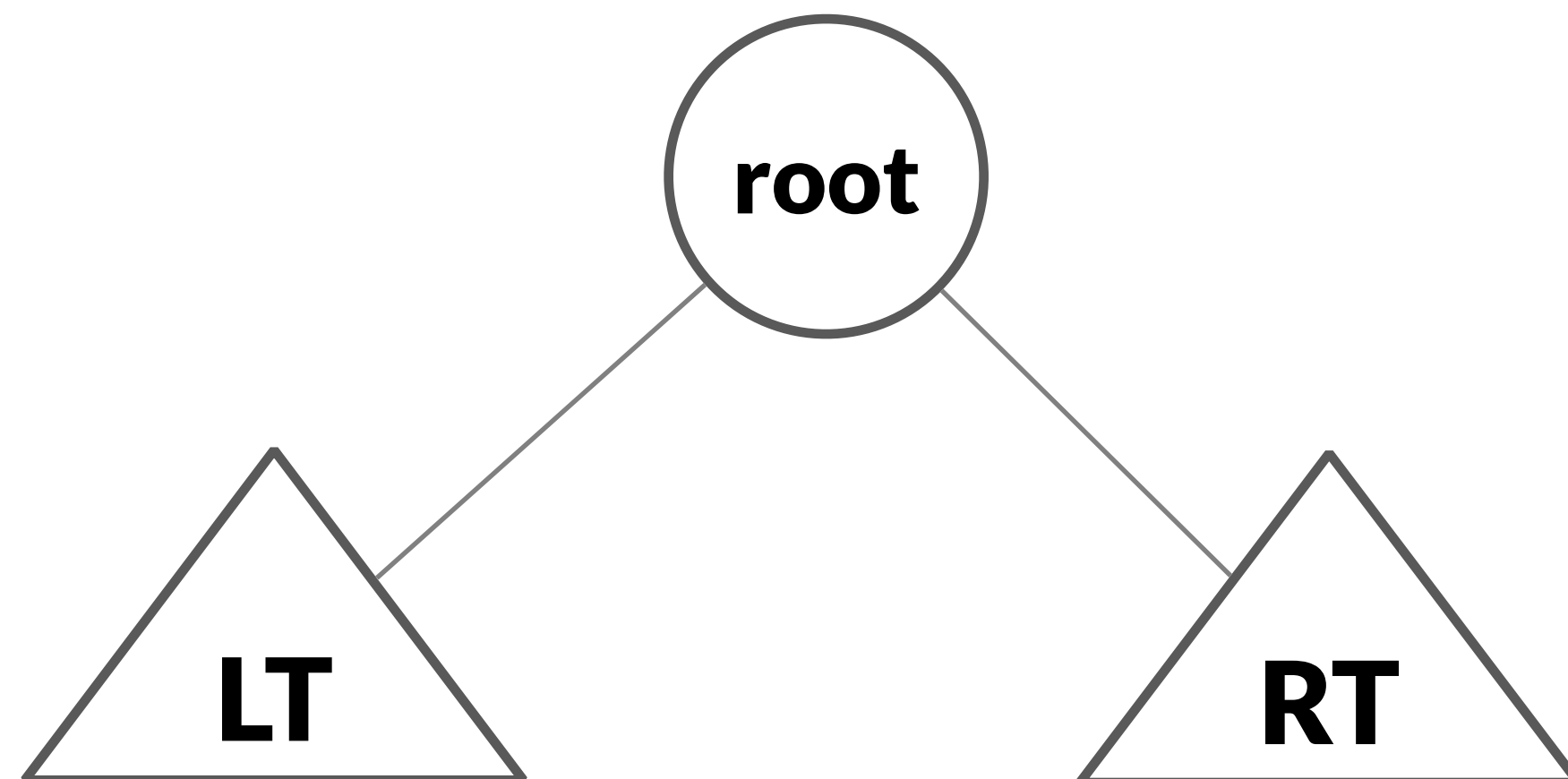
**Contains no nodes, or**

**Is comprised of three disjoint sets of nodes:**

**a root node,**

**a binary tree called its left subtree, and**

**a binary tree called its right subtree**

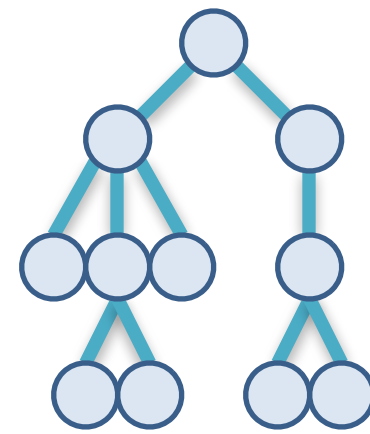




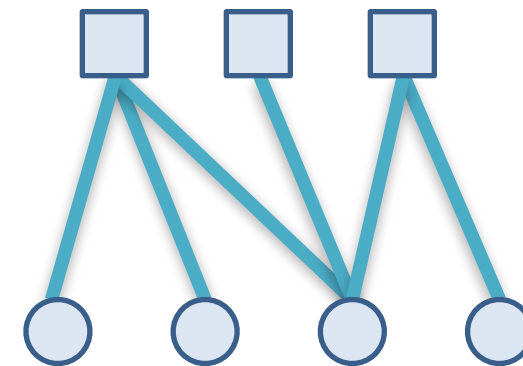
# Different Kinds of Graphs

Over 1000 different graph classes

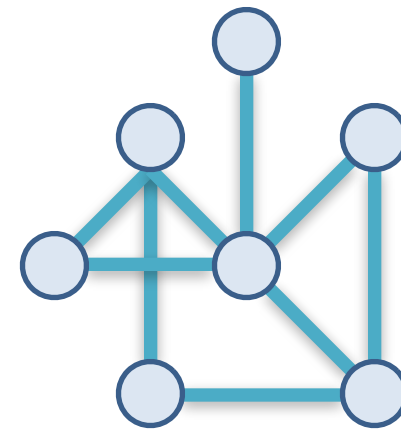
Tree



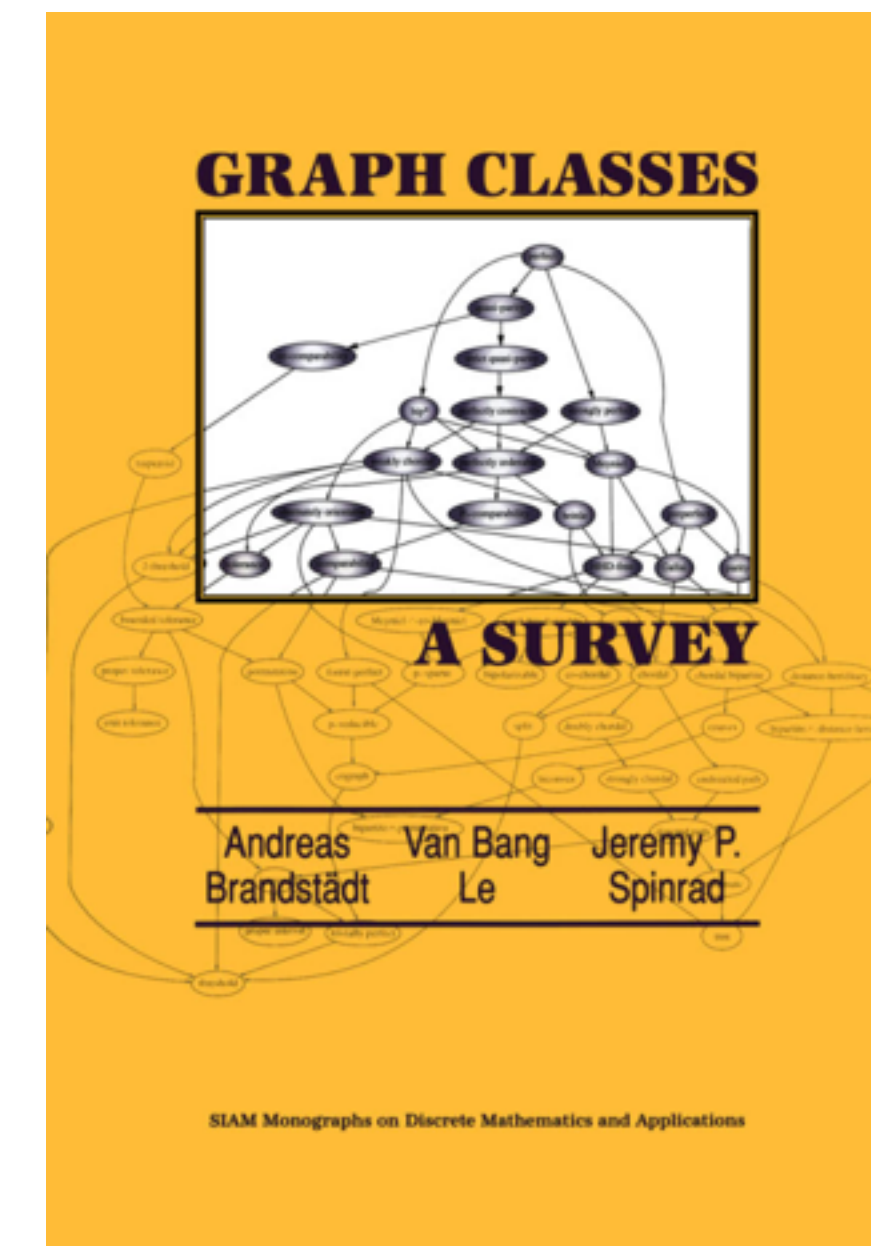
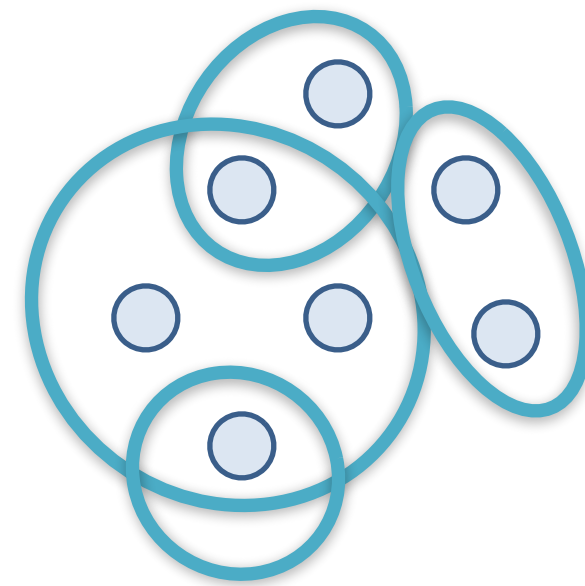
Bipartite Graph



Network



Hypergraph



A. Brandstädt et al. 1999

# Graph Measures

## Node degree $\deg(x)$

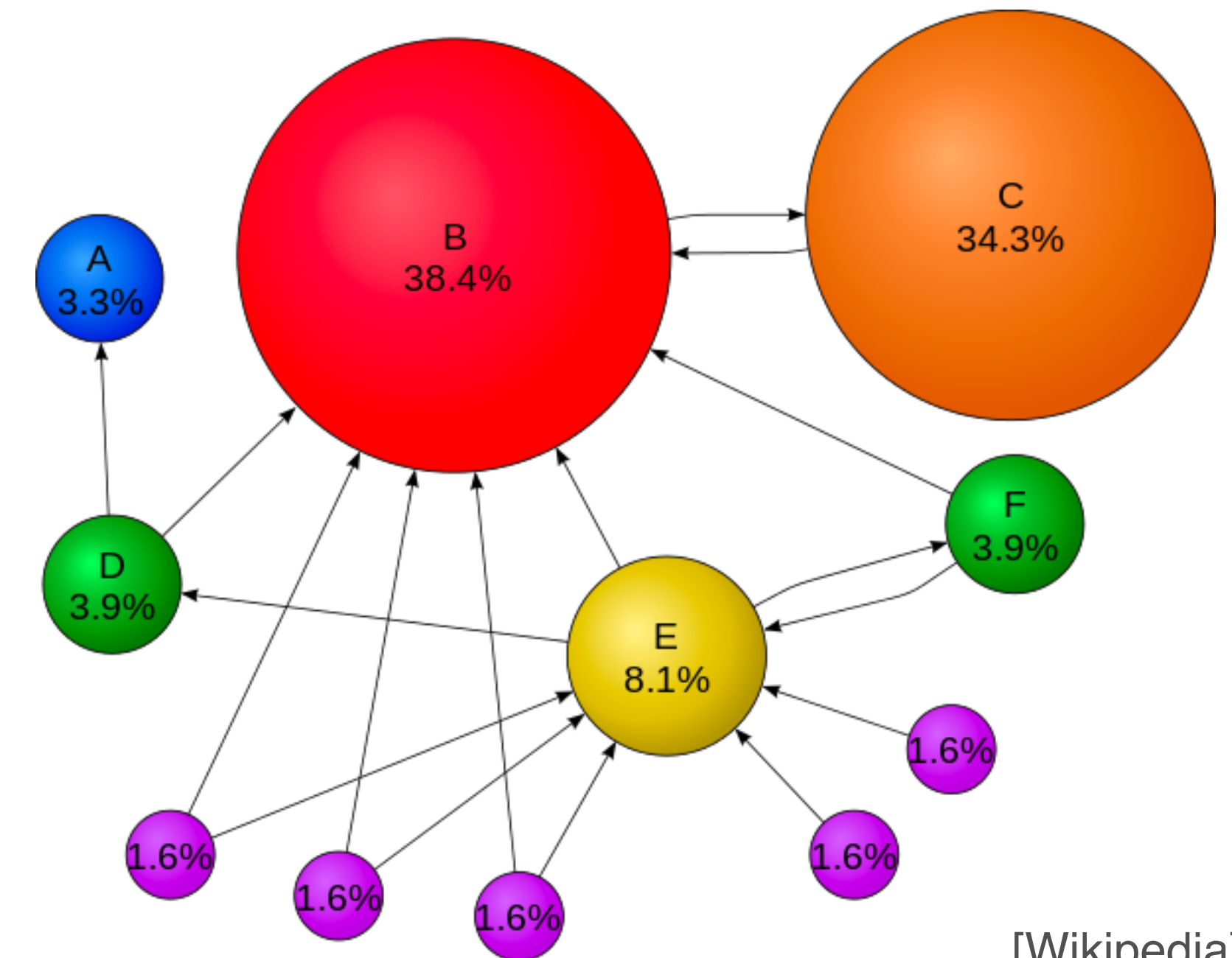
The number of edges being incident to this node. For directed graphs indeg/outdeg are considered separately.

## Diameter of graph $G$

The longest shortest path within  $G$ .

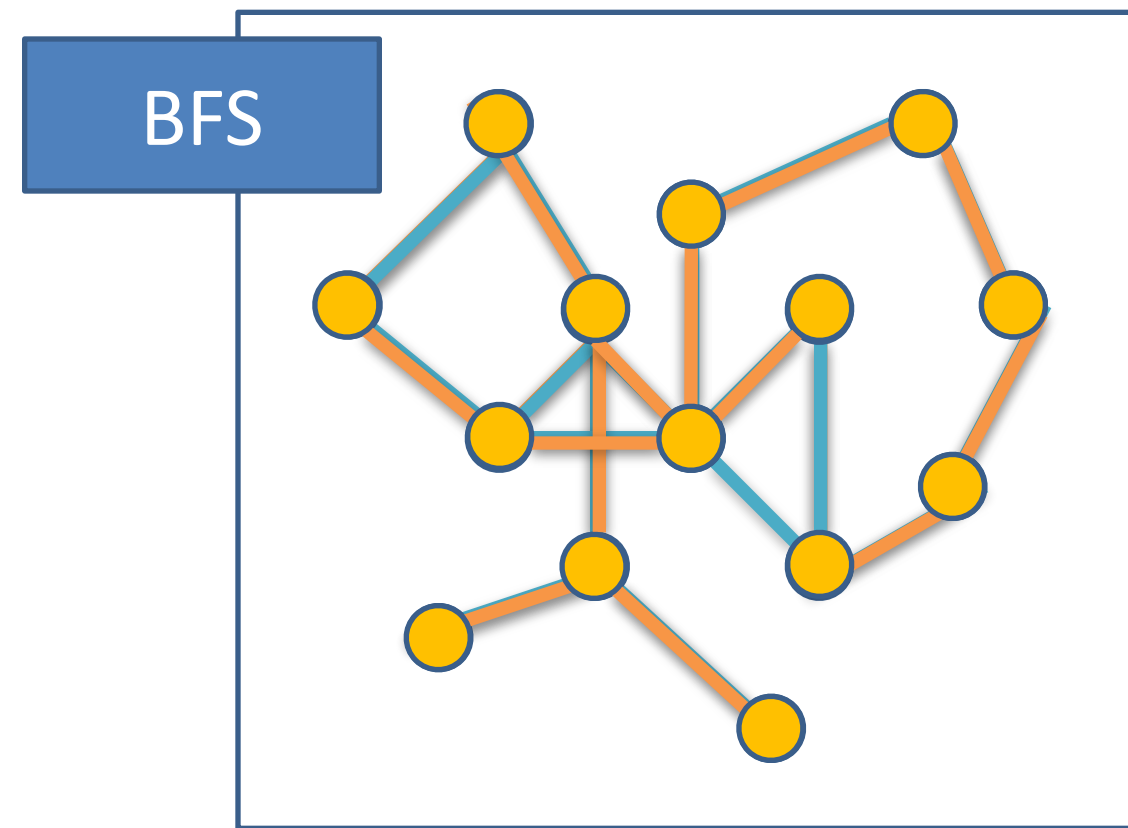
## Pagerank

count number & quality of links

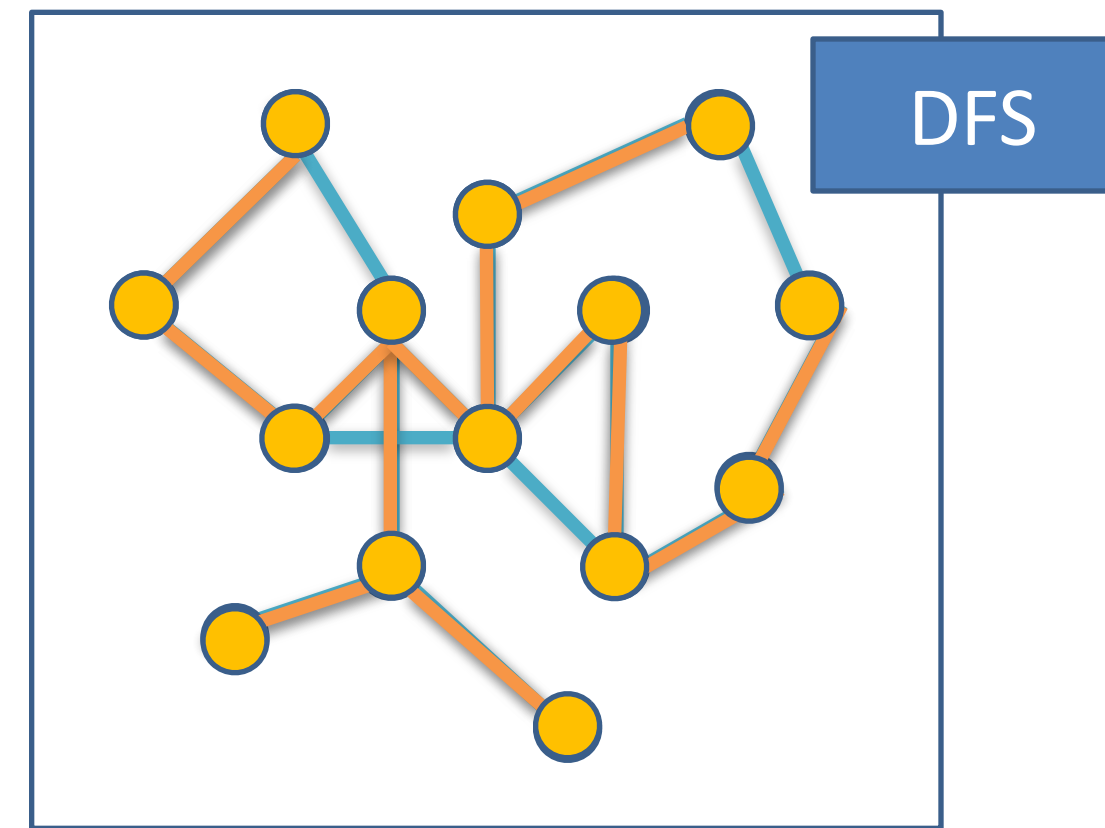


# Graph Algorithms (1)

Traversal: Breadth First Search, Depth First Search



- generates neighborhoods
- hierarchy gets rather wide than deep
- solves single-source shortest paths (SSSP)



- classical way-finding/back-tracking strategy
- tree serialization
- topological ordering



# Hard Graph Algorithms (NP-Complete)

Longest path

Largest clique

Maximum independent set (set of vertices in a graph, no two of which are adjacent)

Maximum cut (separation of vertices in two sets that cuts most edges)

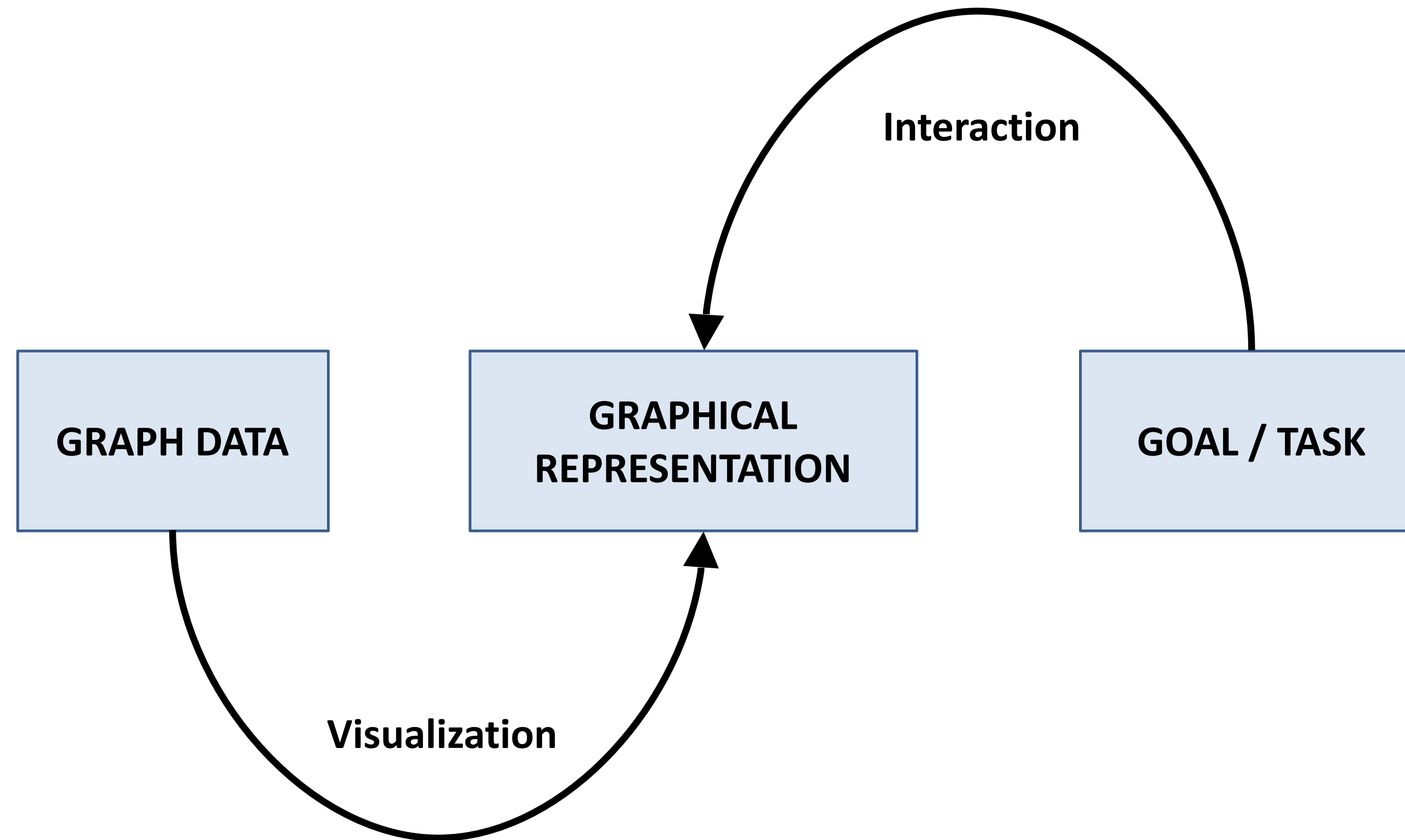
Hamiltonian path/cycle (path that visits all vertexes once)

Coloring / chromatic number (colors for vertices where no adjacent v. have same color)

Minimum degree spanning tree

# Graph and Tree Visualization

# Setting the Stage



How to decide which **representation** to use for which **type of graph** in order to achieve which kind of **goal**?



# Different Kinds of Tasks/Goals

Two principal types of tasks: **attribute-based (ABT)** and **topology-based (TBT)**

**Localize** – find a single or multiple nodes/edges that fulfill a given property

- ABT: Find the edge(s) with the maximum edge weight.
- TBT: Find all adjacent nodes of a given node.

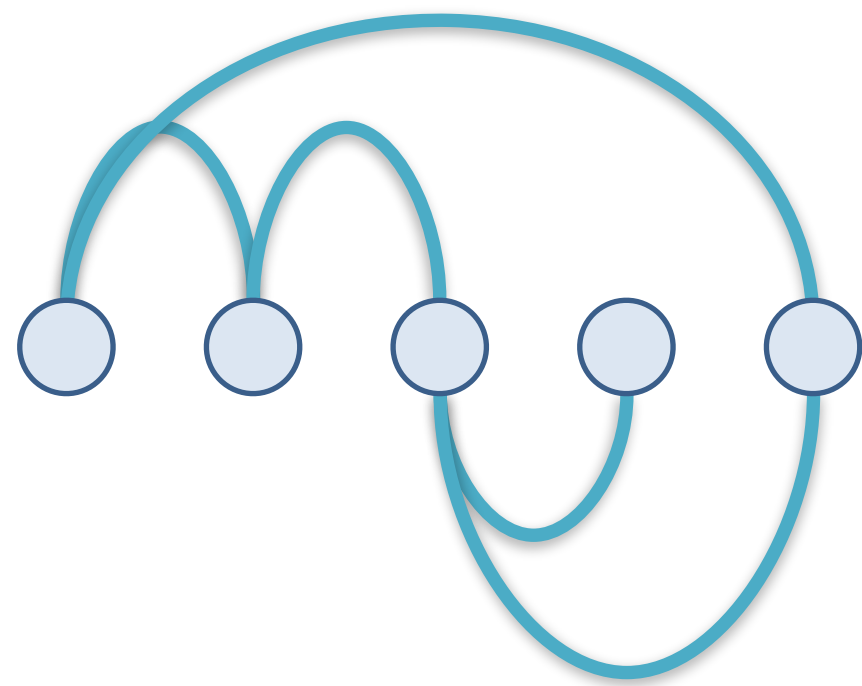
**Quantify** – count or estimate a numerical property of the graph

- ABT: Give the number of all nodes.
- TBT: Give the indegree (the number of incoming edges) of a node.

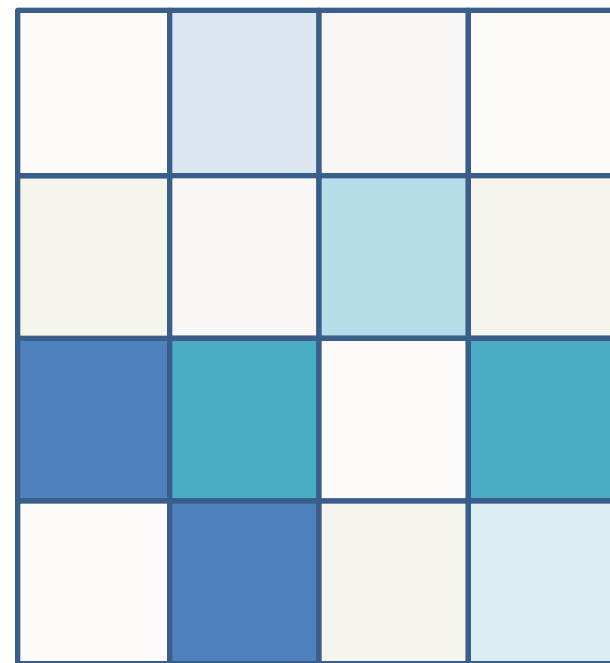
**Sort/Order** – enumerate the nodes/edges according to a given criterion

- ABT: Sort all edges according to their weight.
- TBT: Traverse the graph starting from a given node.

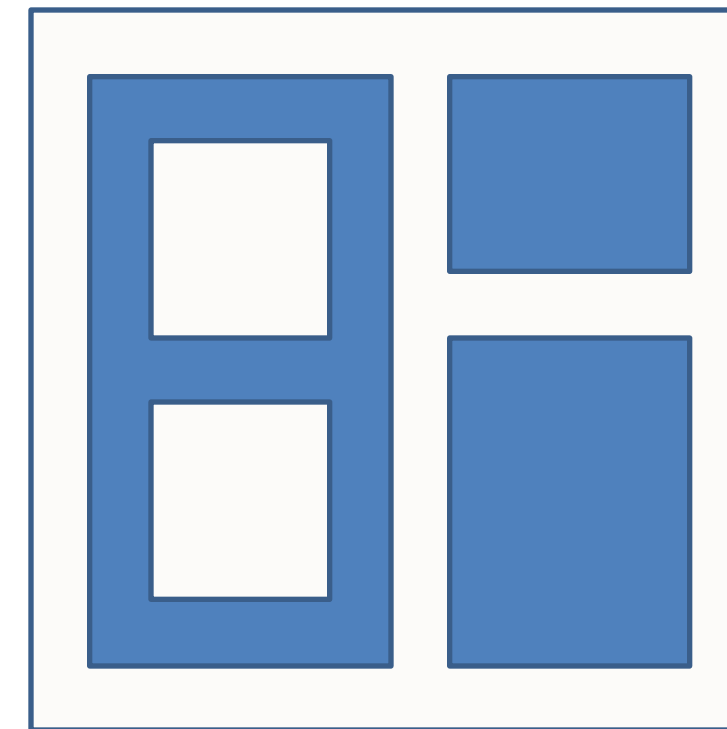
# Three Types of Graph Representations



Explicit  
(Node-Link)



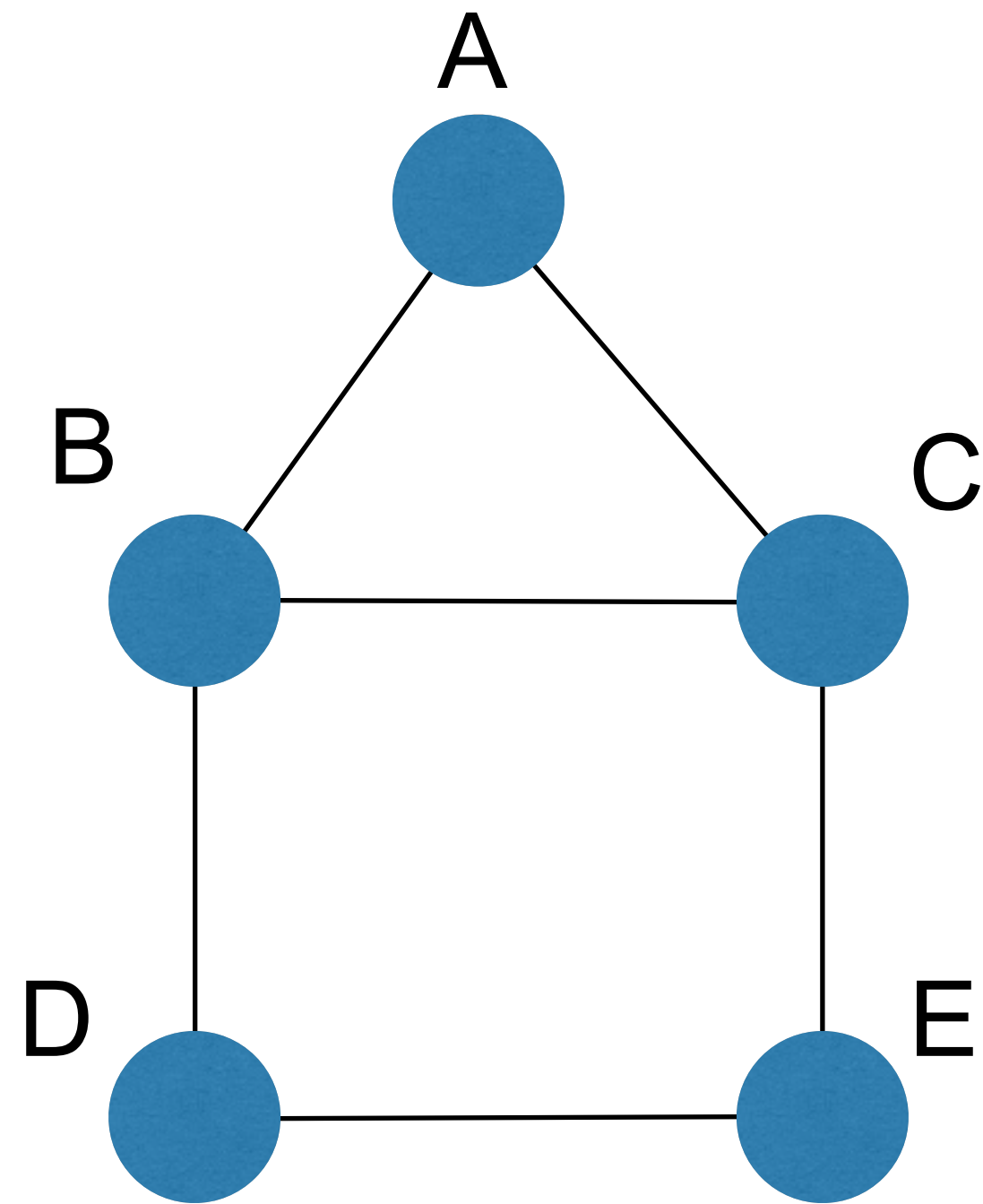
Matrix



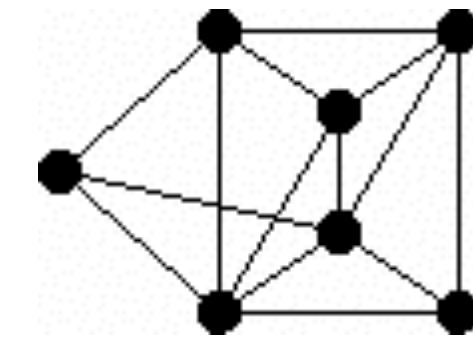
Implicit

# Explicit Graph Representations

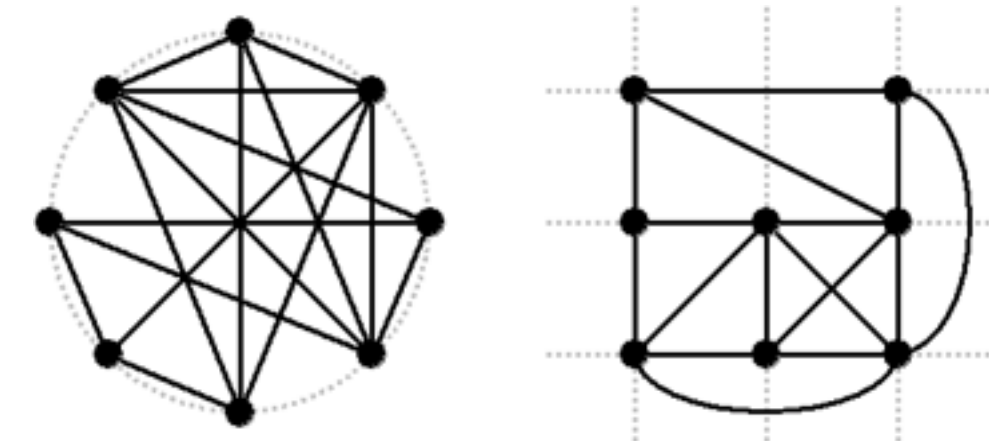
Node-link diagrams: vertex = point, edge = line/arc



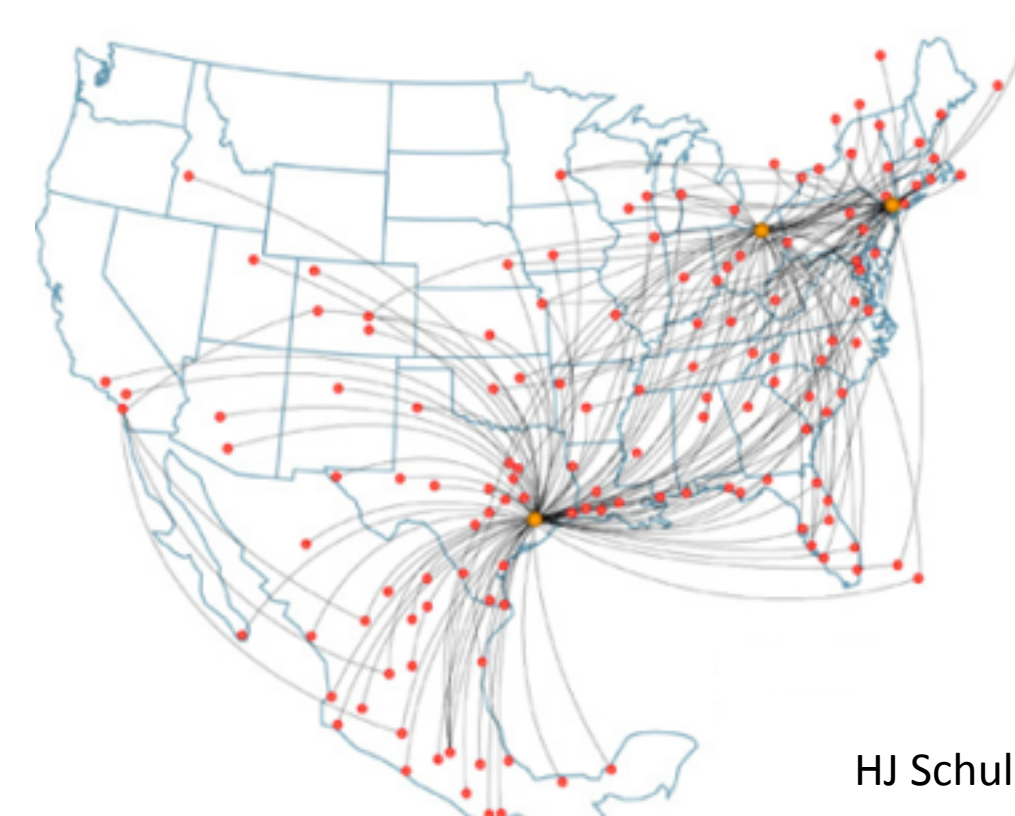
**Free**



**Styled**



**Fixed**





# Criteria for Good Node-Link Layout

Minimized **edge crossings**

Minimized **distance** of neighboring nodes

Minimized **drawing area**

Uniform edge **length**

Minimized edge **bends**

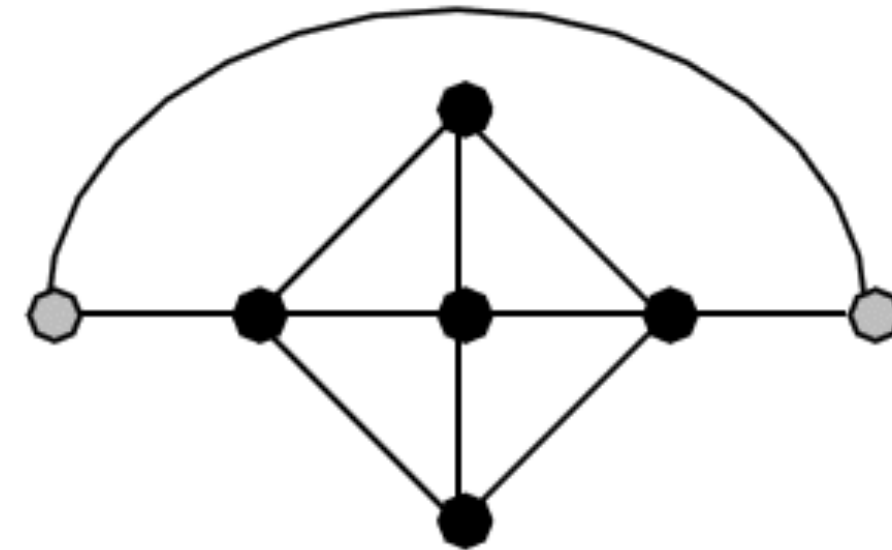
Maximized **angular distance** between different edges

Aspect ratio about 1 (not too long and not too wide)

**Symmetry**: similar graph structures should look similar

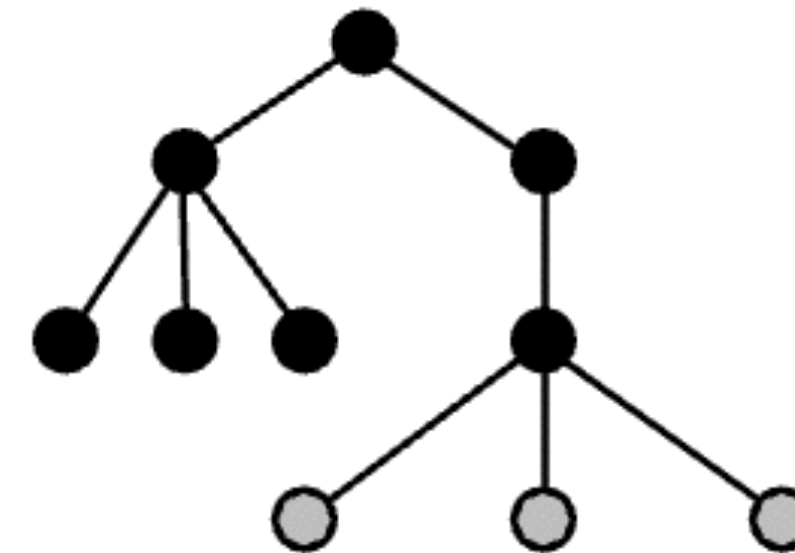
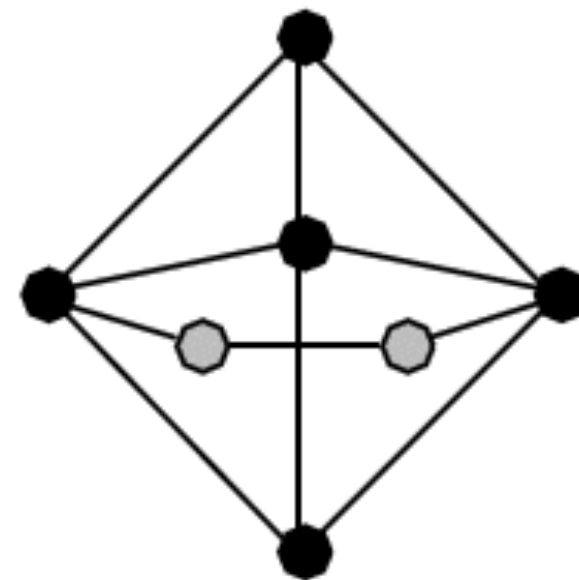
# Conflicting Criteria

Minimum number  
of edge crossings



vs.

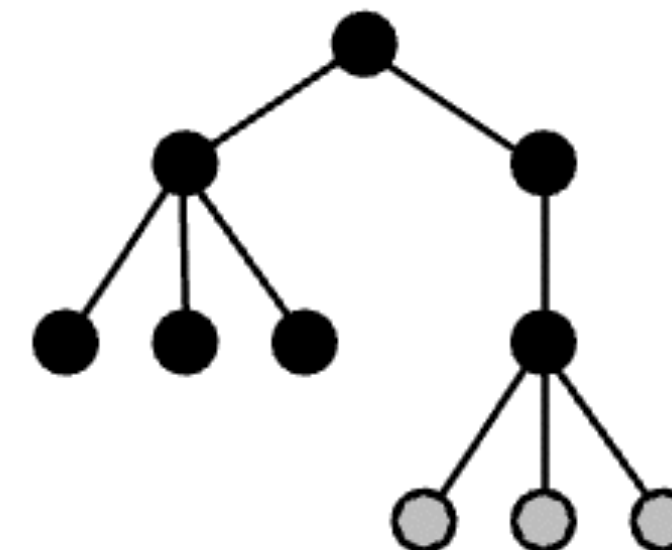
Uniform edge  
length



Space utilization

vs.

Symmetry





# Force Directed Layouts

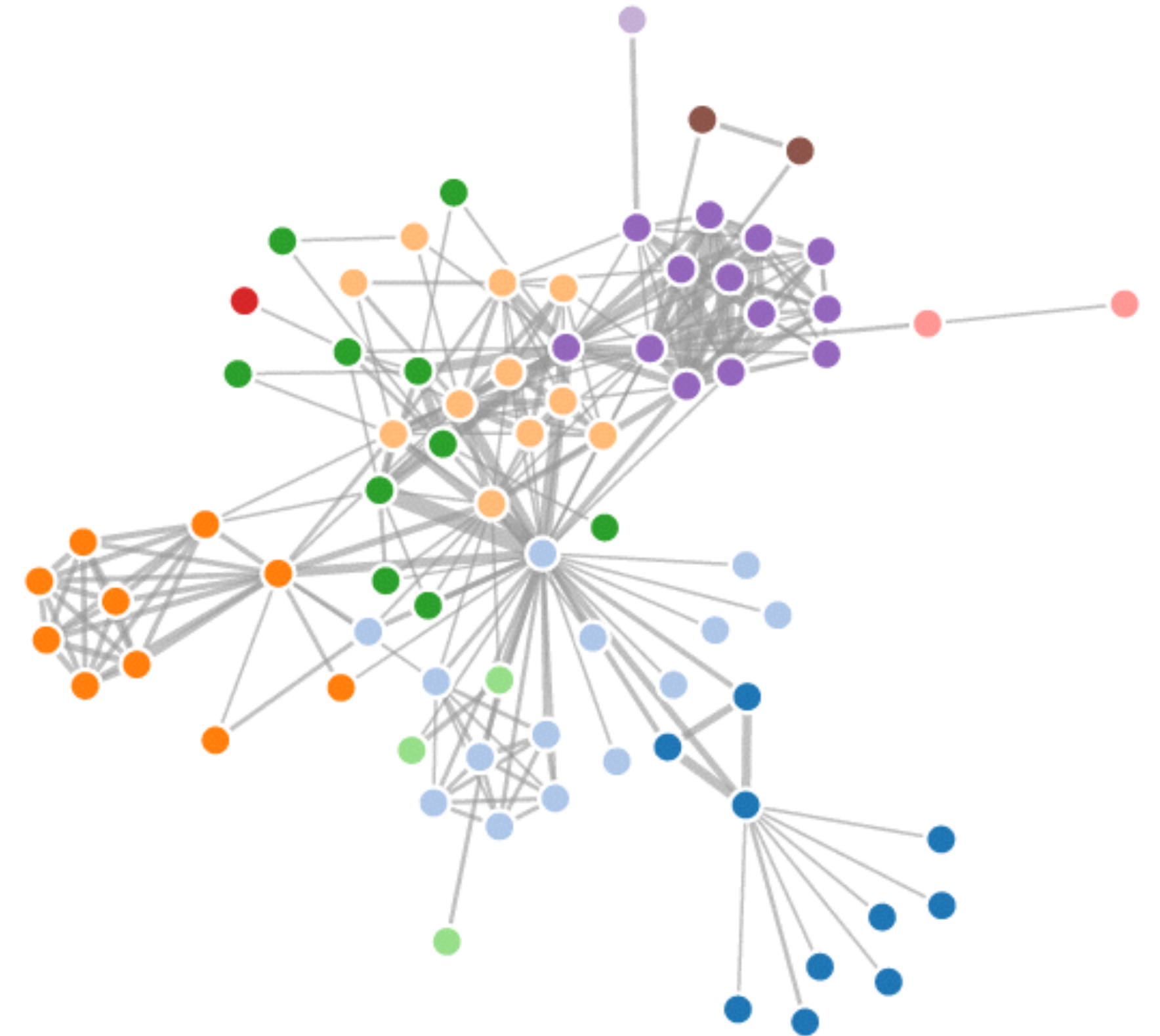
Physics model:  
edges = springs,  
vertices = repulsive magnets  
in practice: damping

Computationally  
expensive:  $O(n^3)$

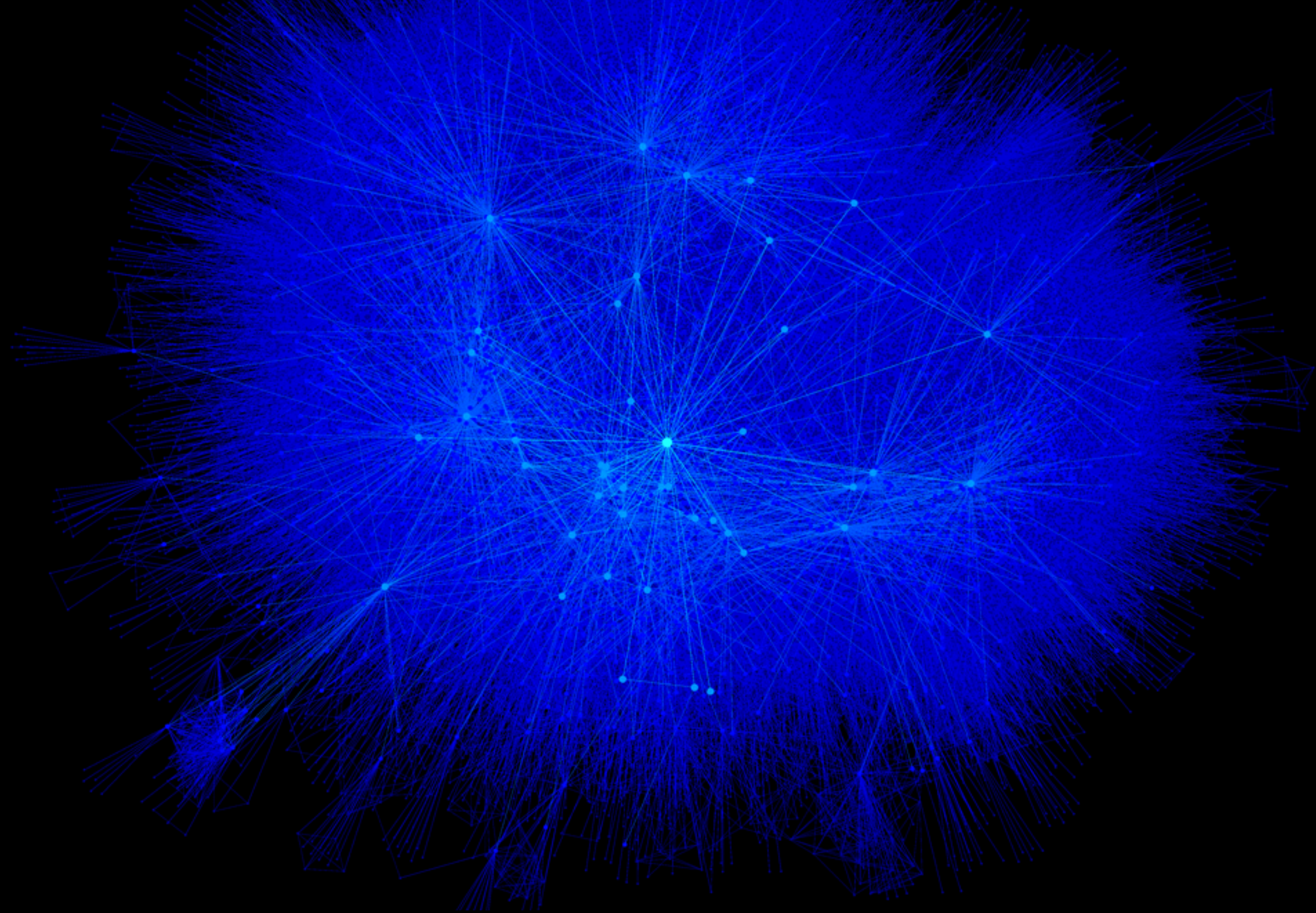
Limit (interactive): ~1000 nodes

Expander  
(pushing nodes apart)

Spring Coil  
(pulling nodes together)



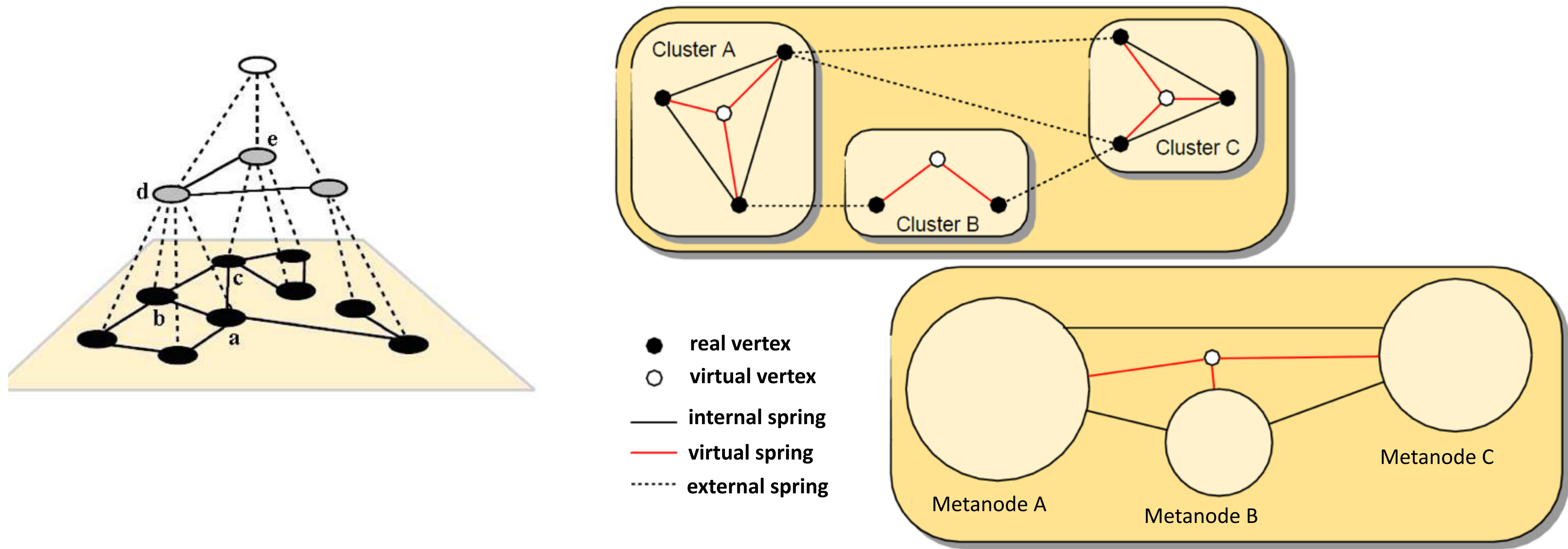




**Giant Hairball**

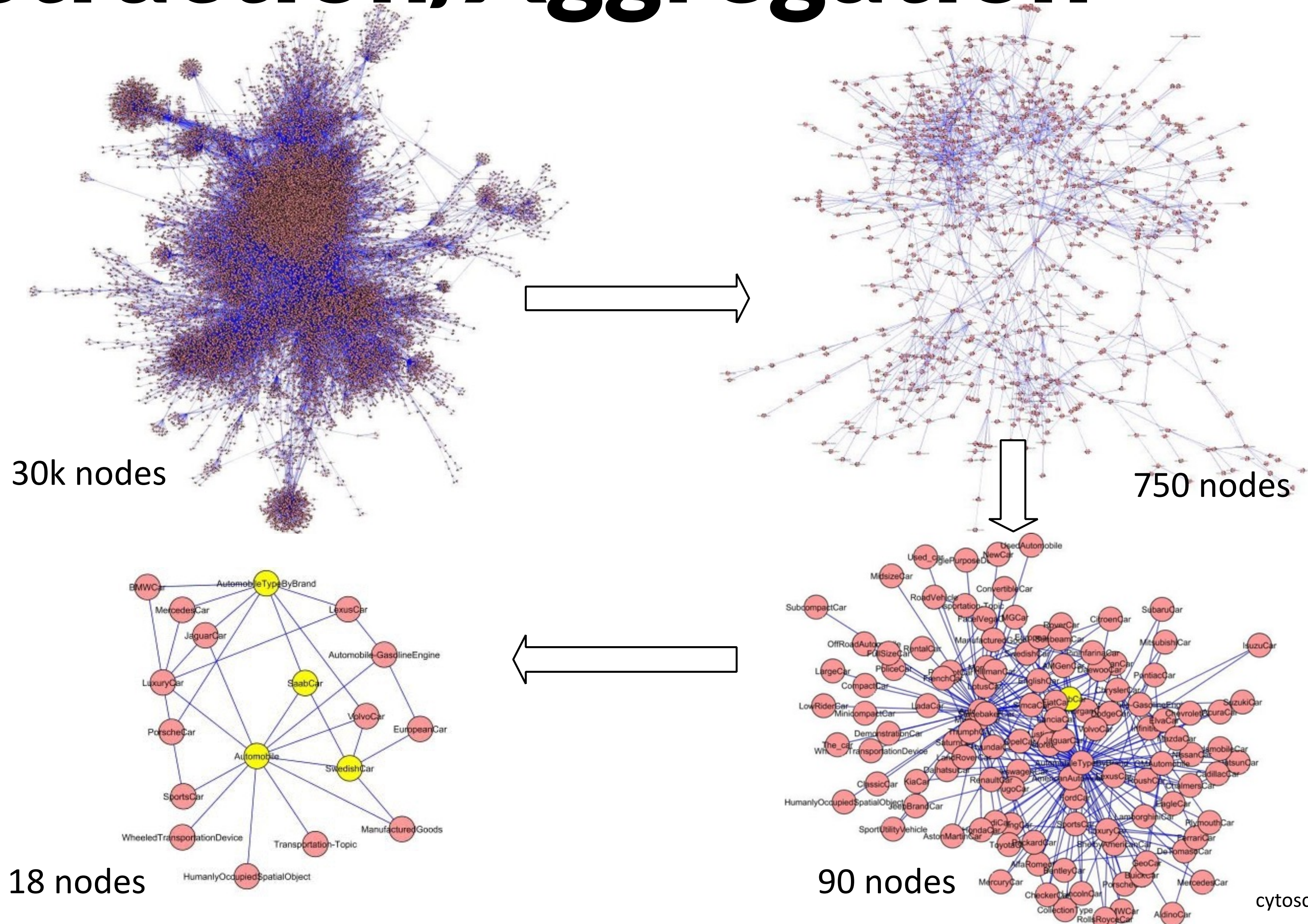


# Address Computational Scalability: Multilevel Approaches





# Abstraction/Aggregation

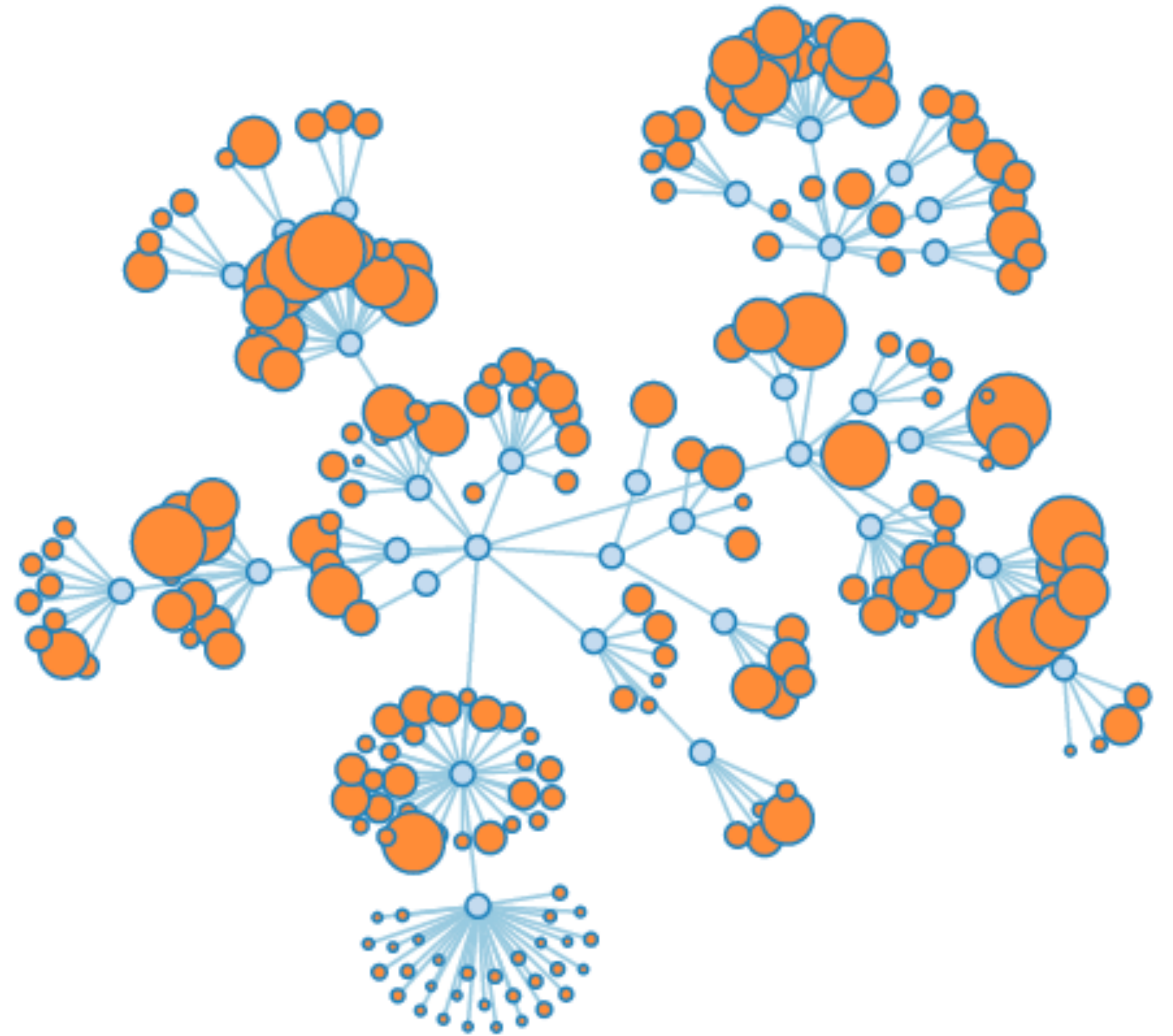




# Collapsible Force Layout

Supernodes: aggregate of  
nodes

manual or algorithmic  
clustering



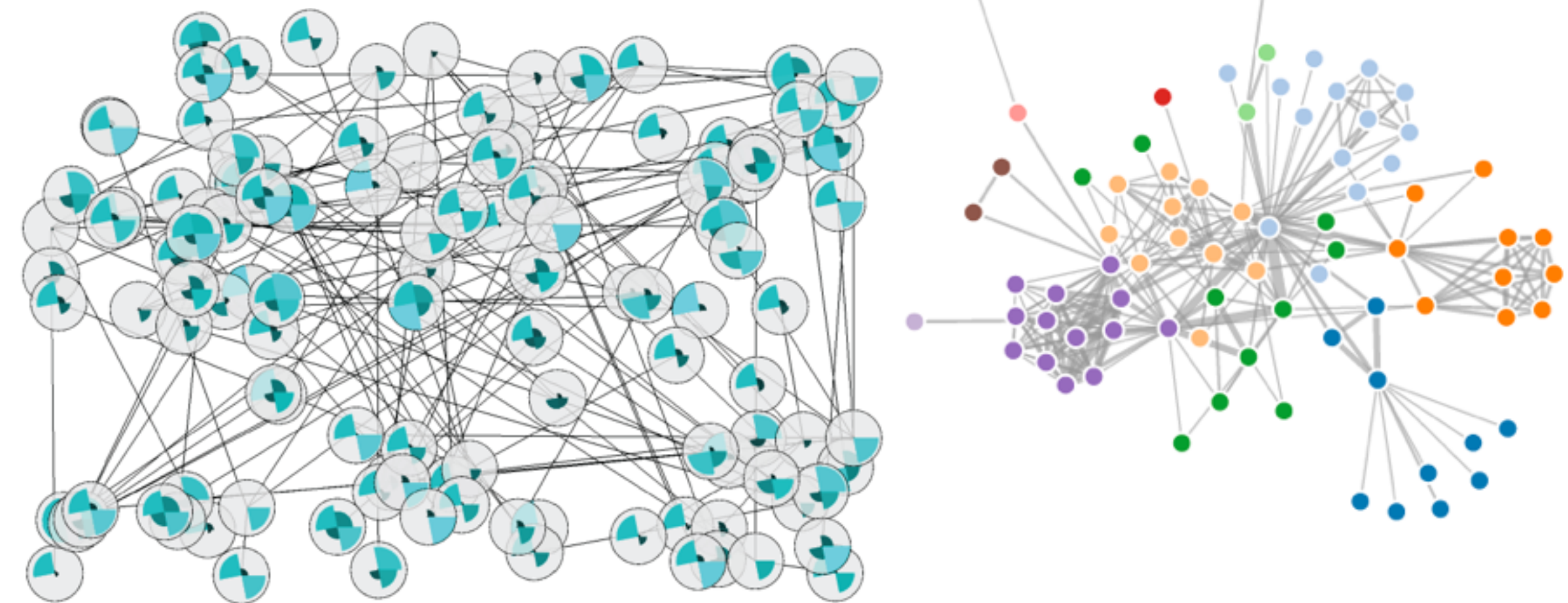
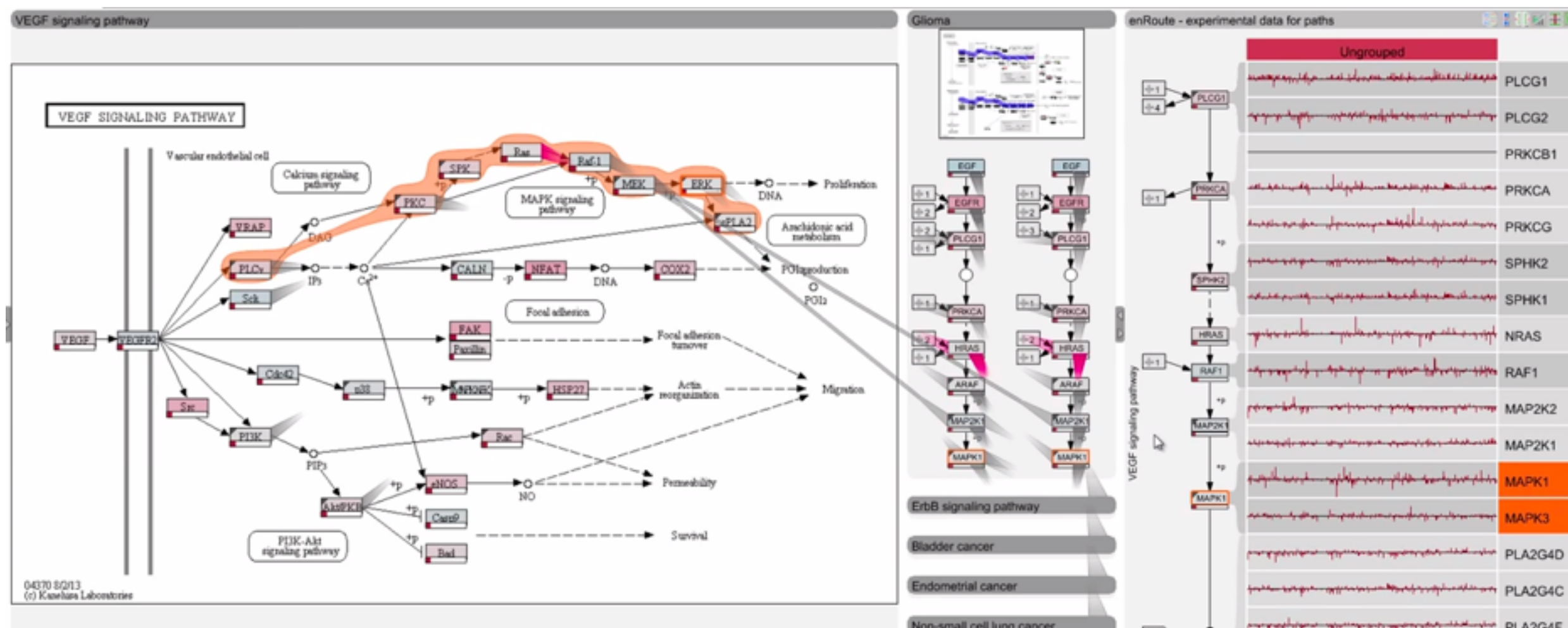
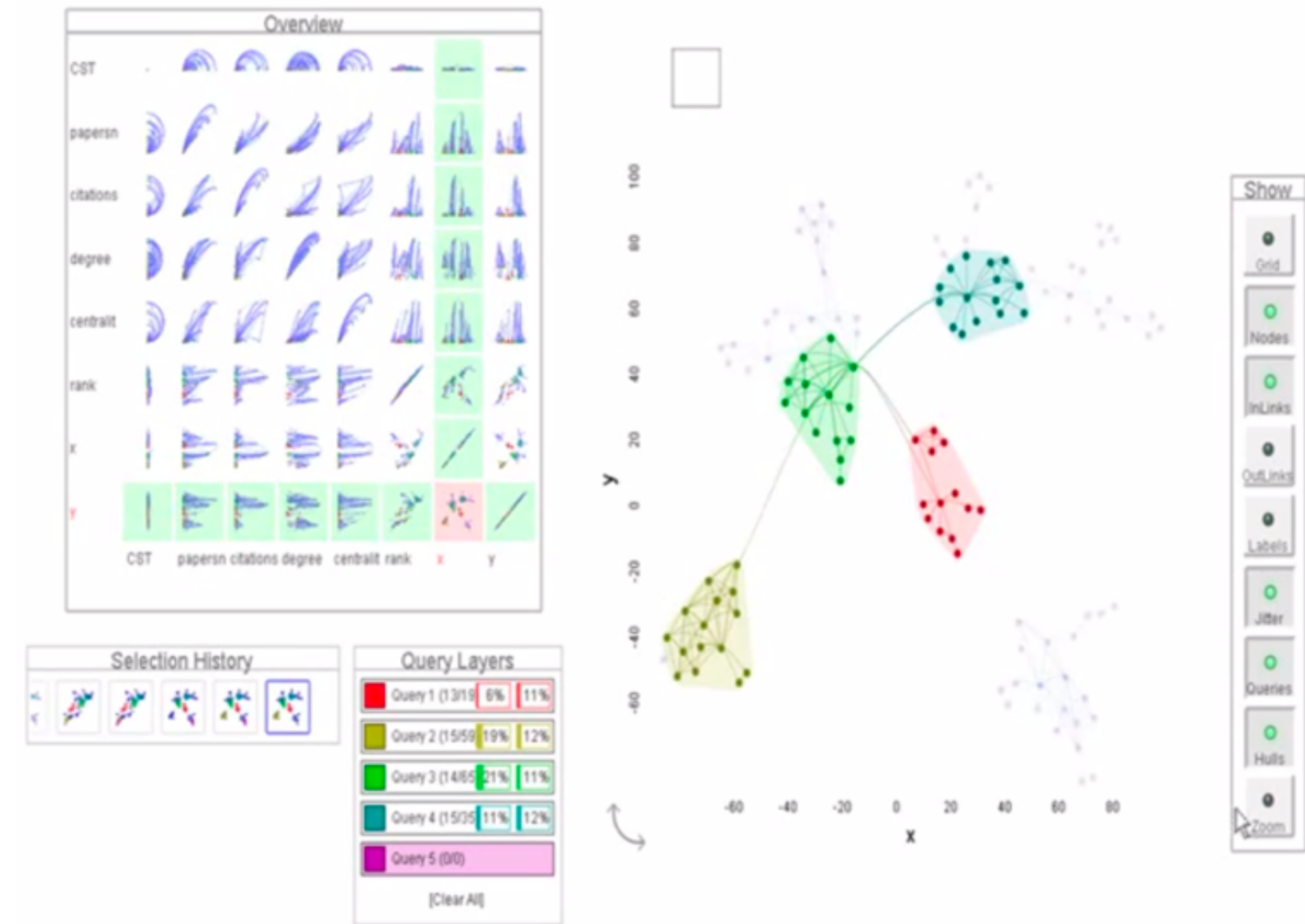


# Node Attributes

Coloring

Position

Multiple Views /  
Path extraction





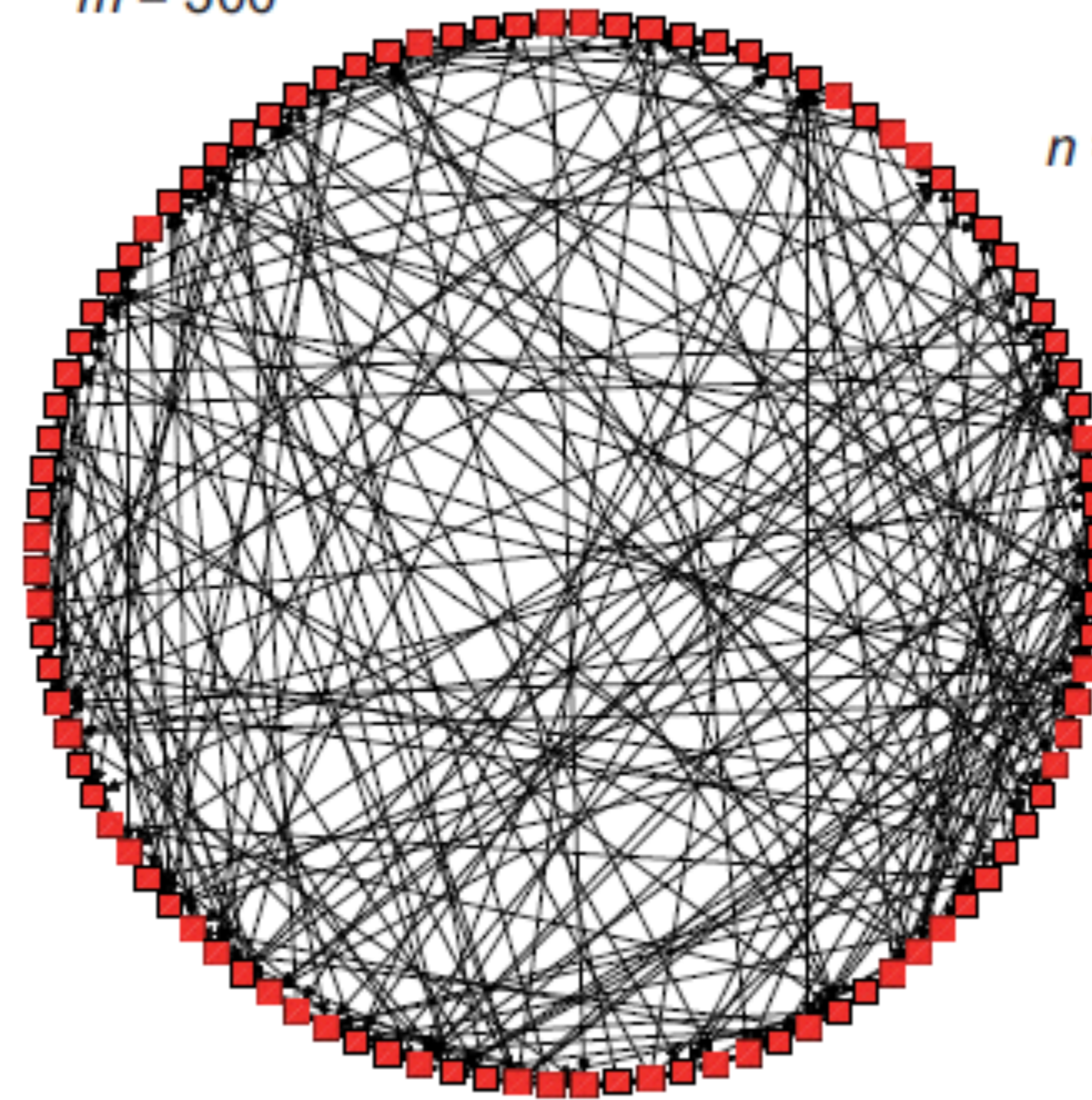
# Styled / Restricted Layouts

Circular Layout

Node ordering

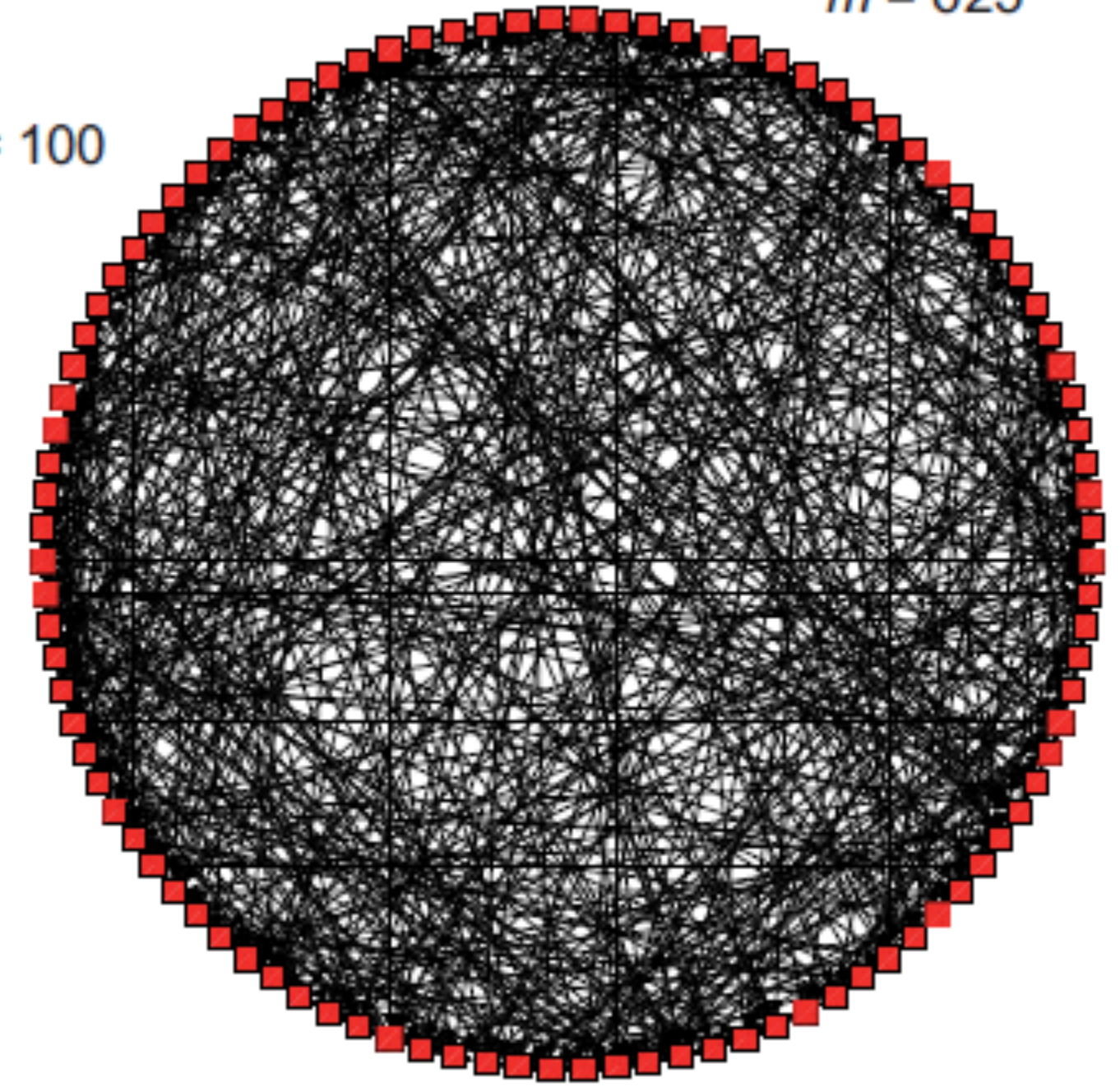
Edge Clutter

$m = 300$



ca. 3% of all possible edges

$m = 625$

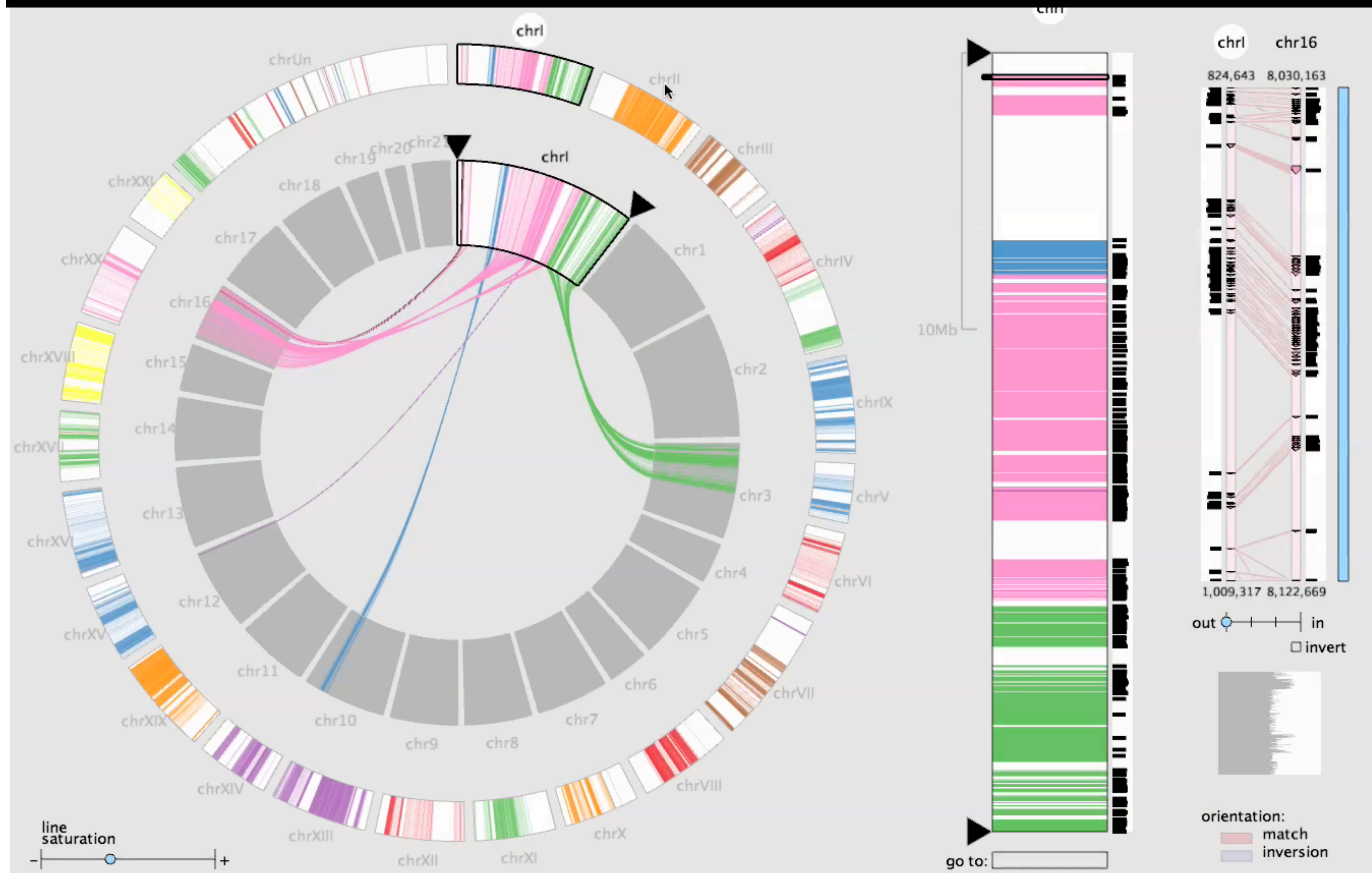


ca. 6,3% of all possible edges



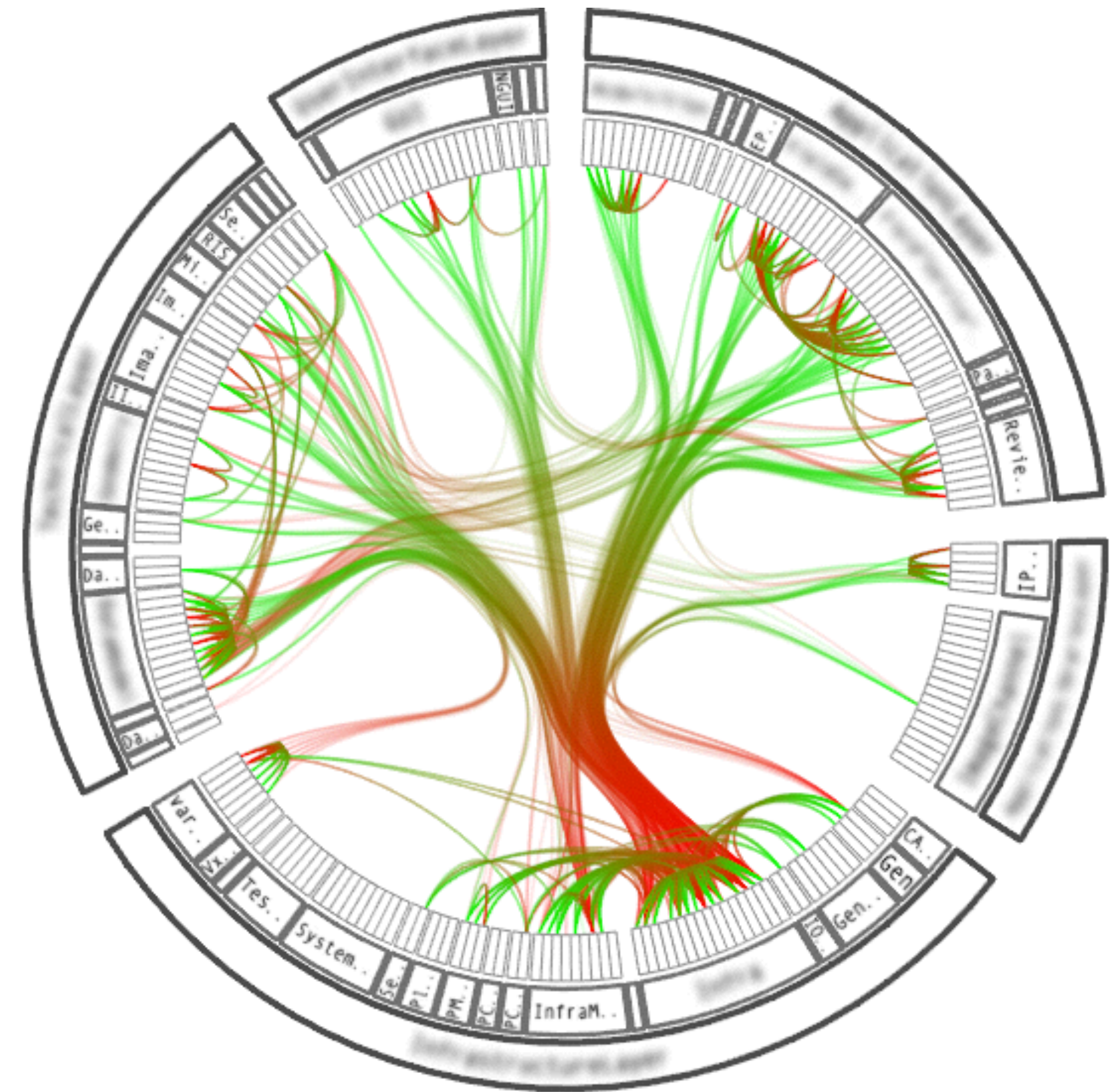
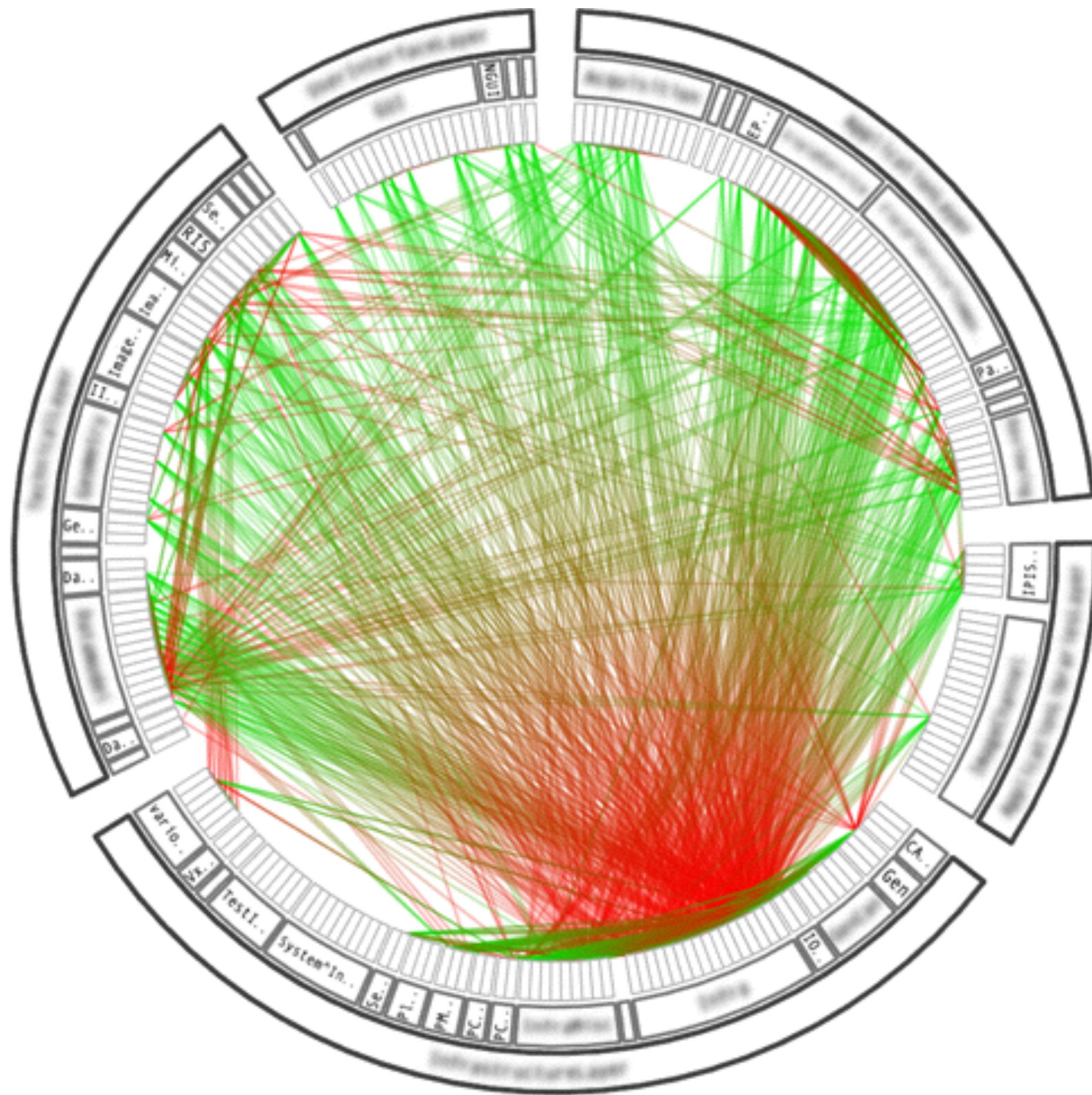
# Example: MizBee

[Meyer et al. 2009]



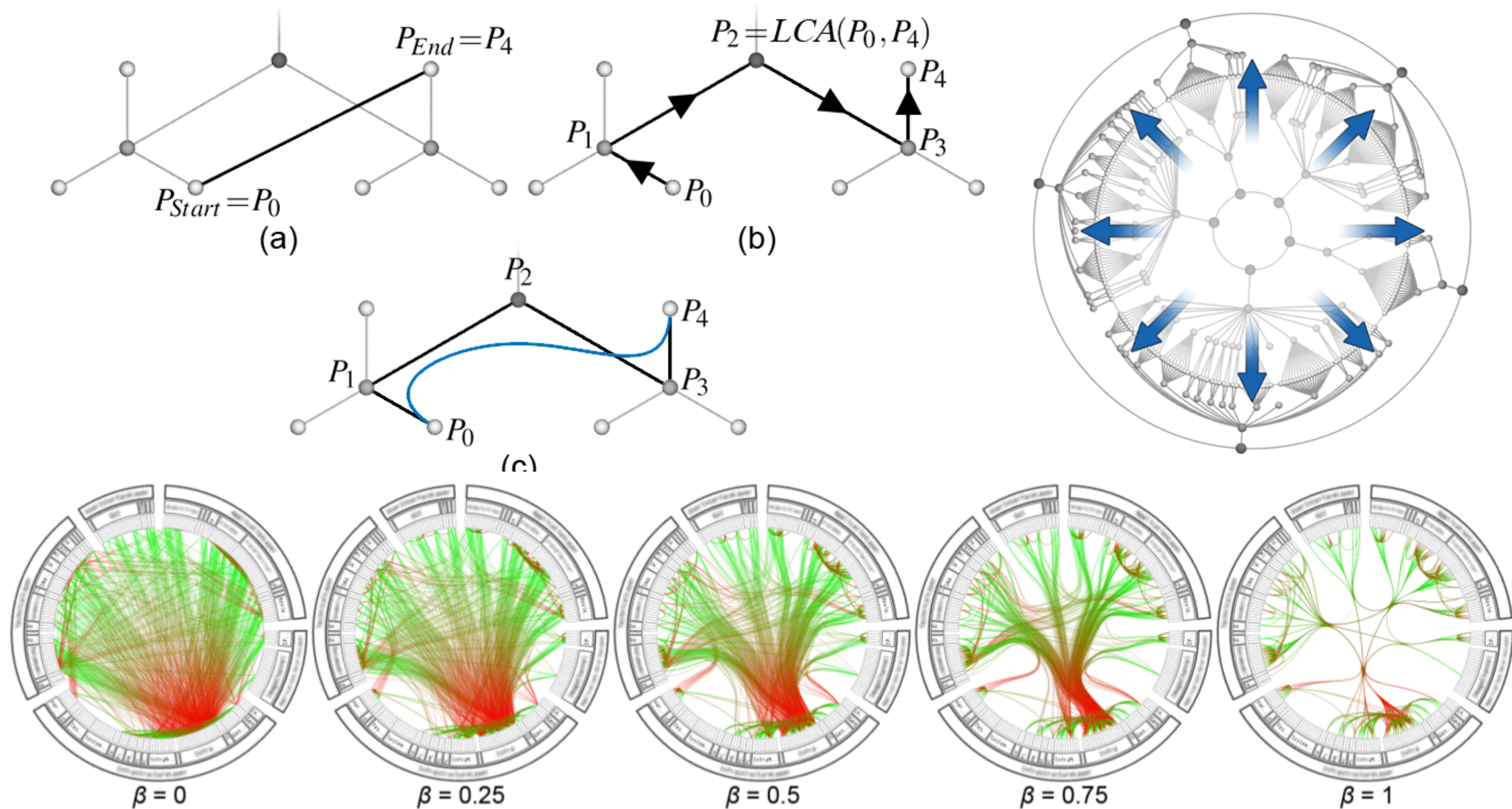


# Reduce Clutter: Edge Bundling





# Hierarchical Edge Bundling



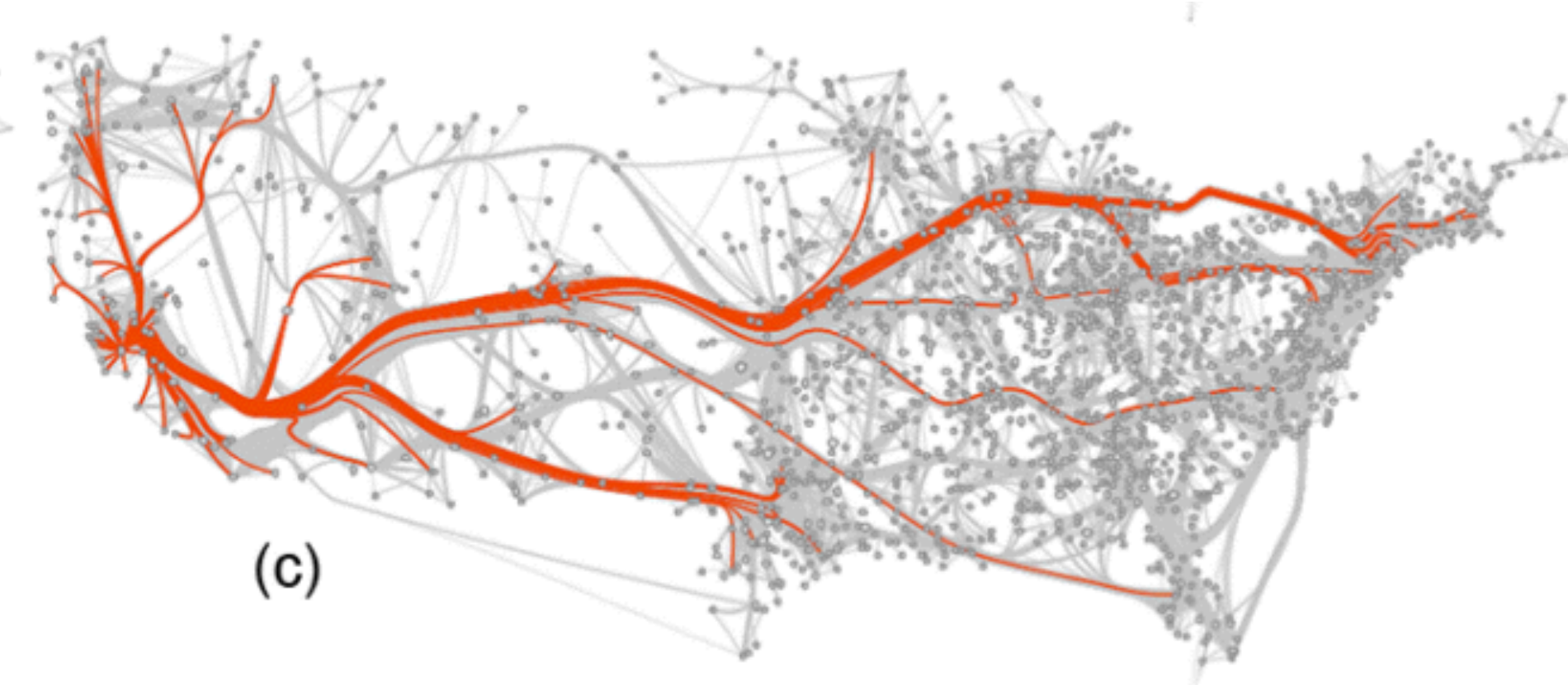
Bundling Strength



# Fixed Layouts

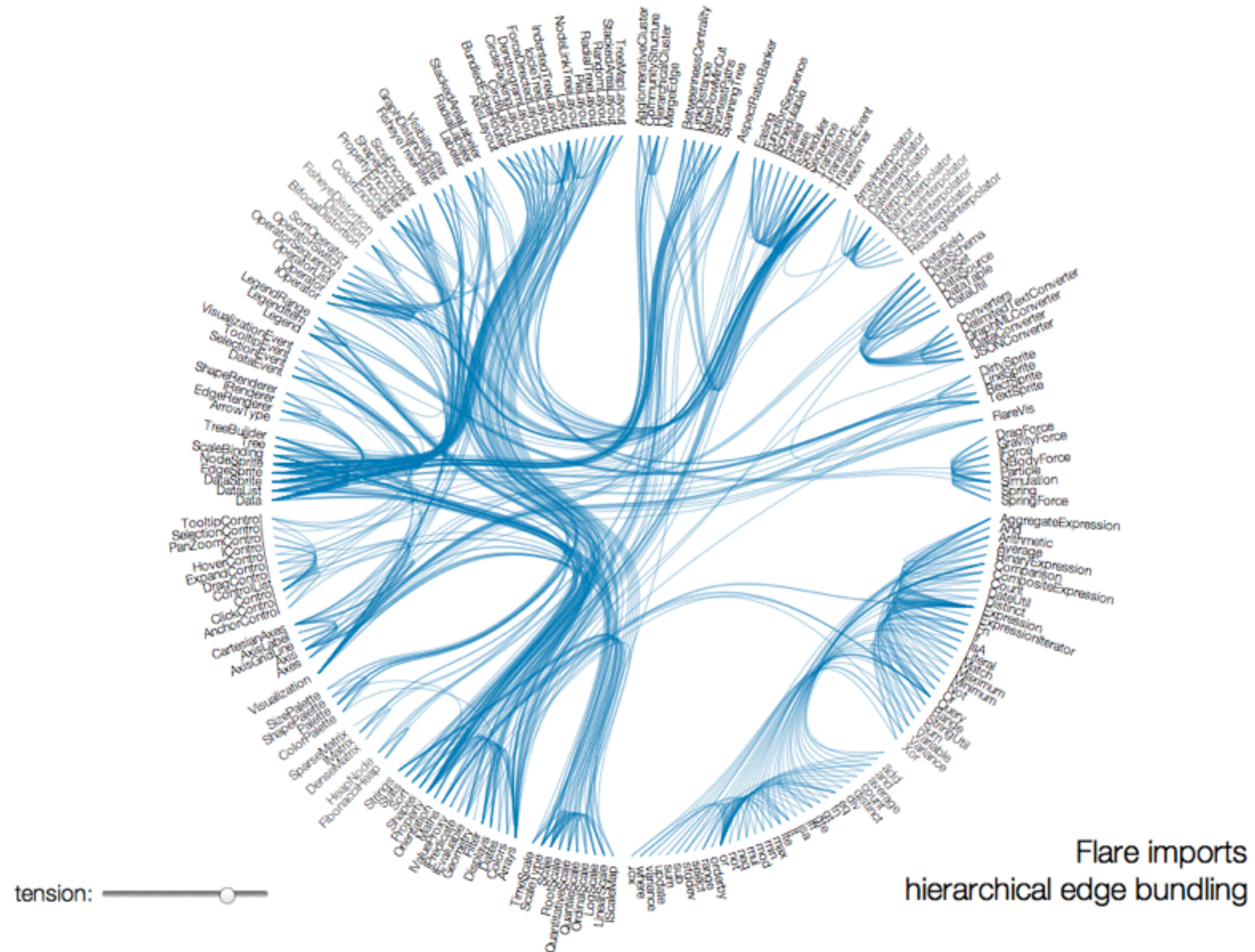
Can't vary position of nodes

Edge routing important





# Bundling Strength

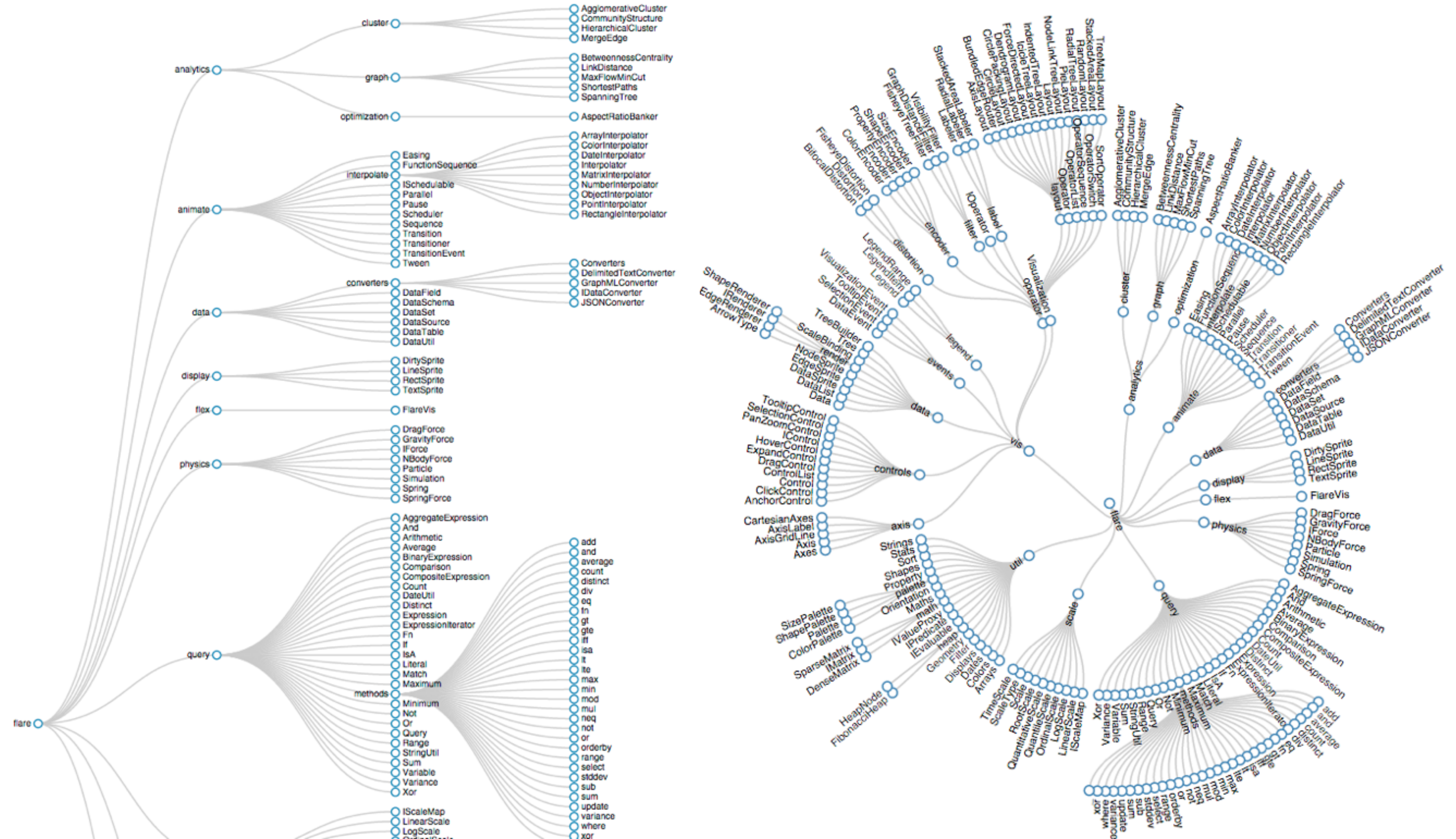




# Explicit Tree Visualization

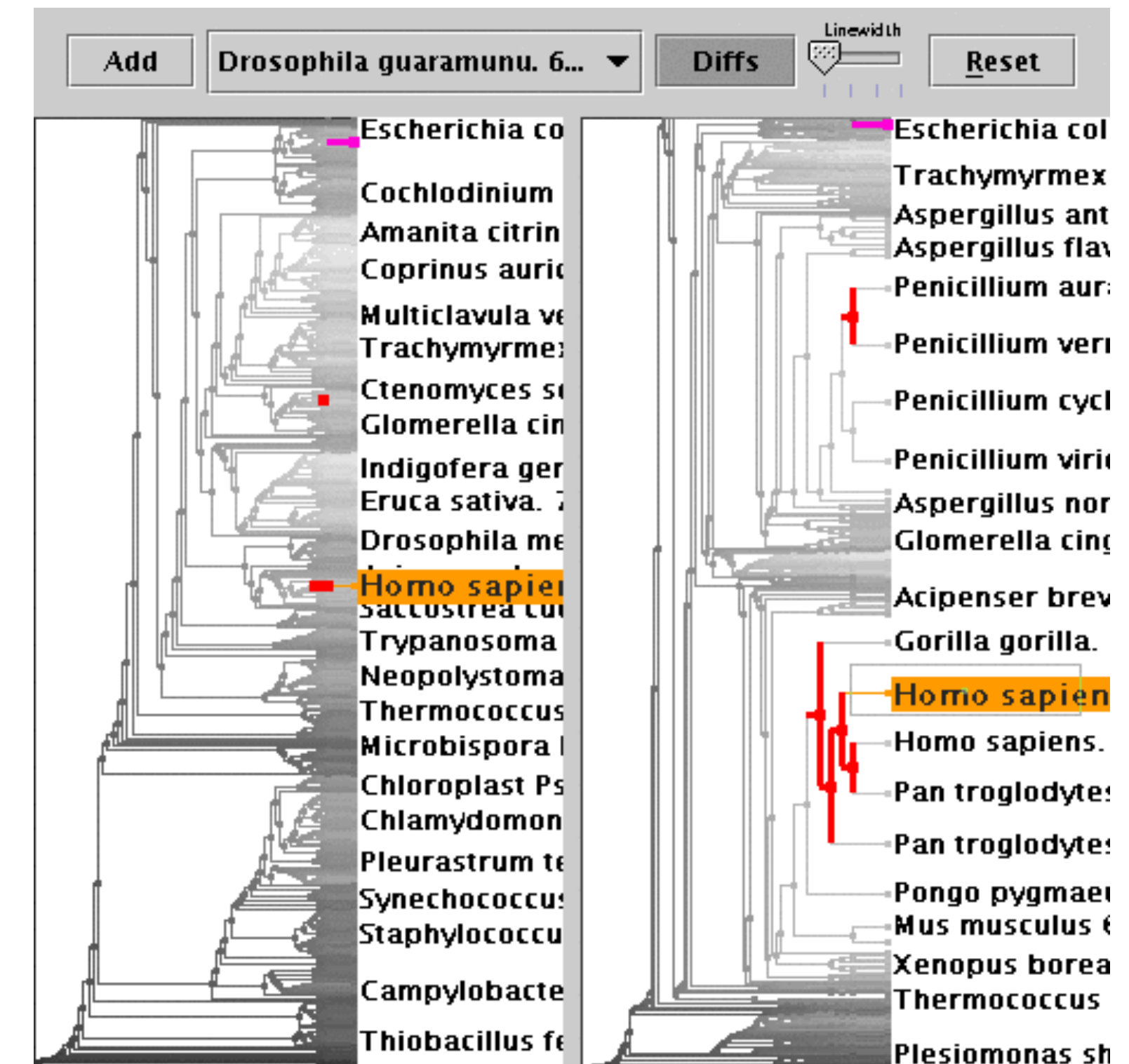
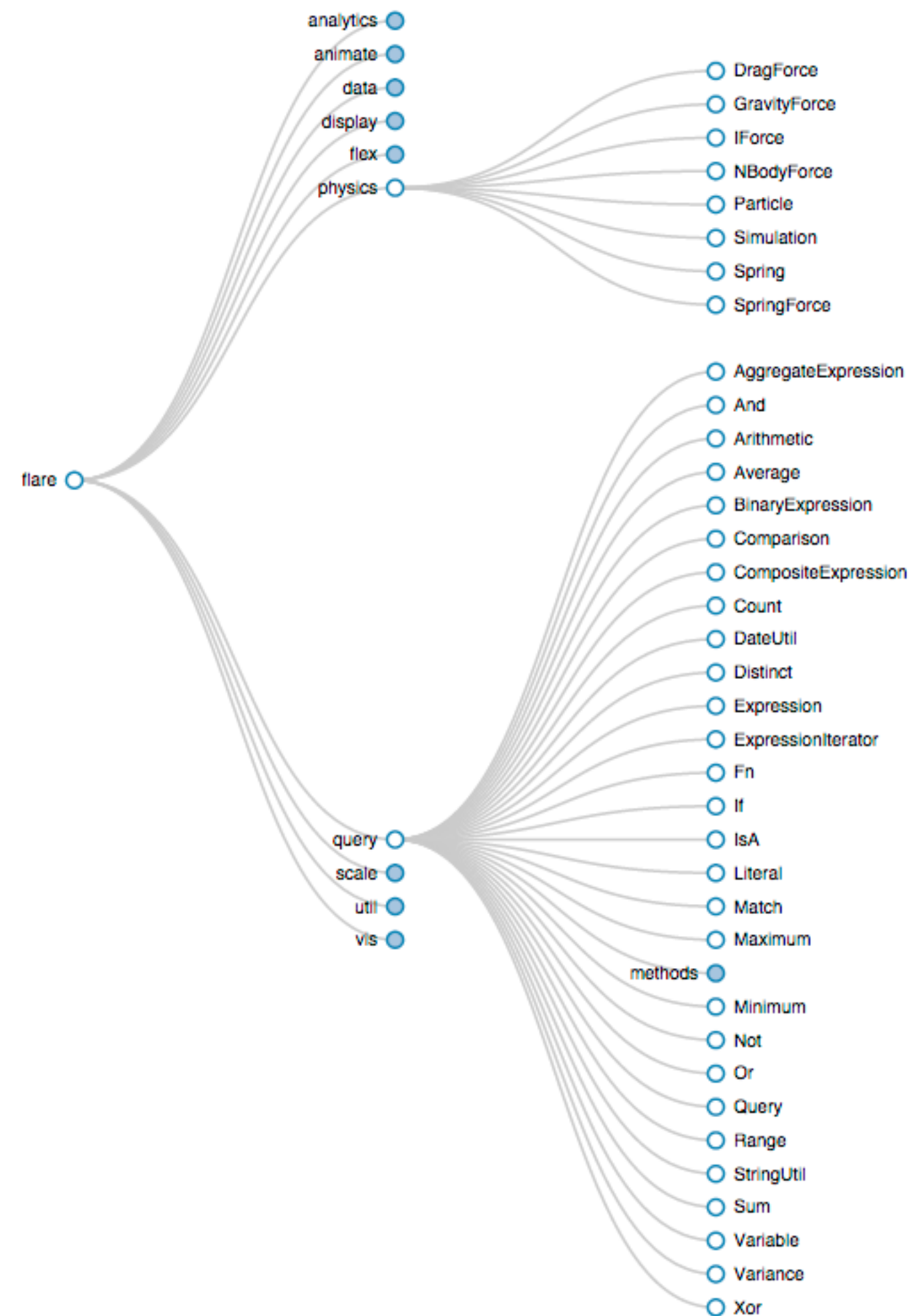
Reingold–  
Tilford layout

<http://billmill.org/pymag-trees/>



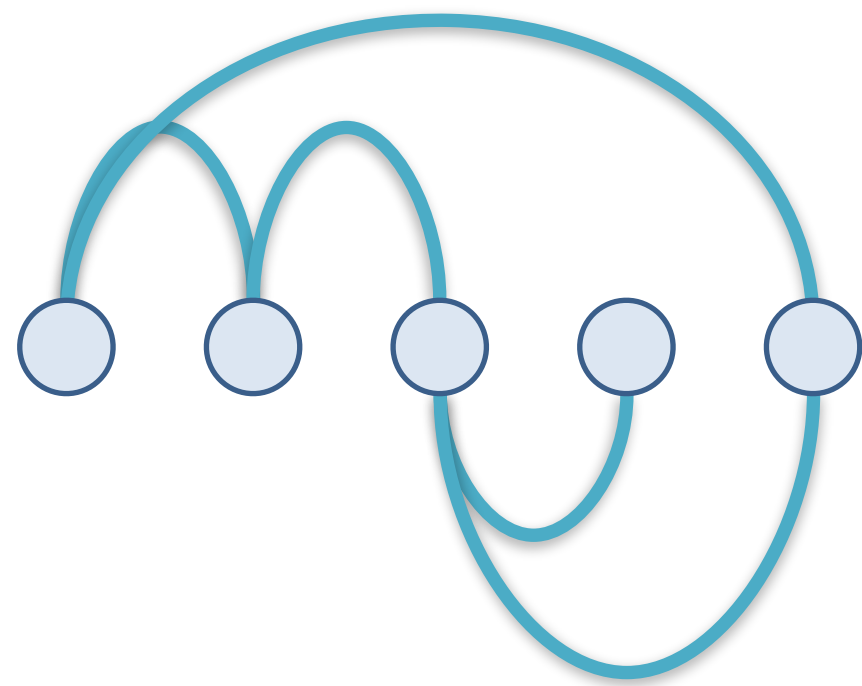


# Tree Interaction, Tree Comparison

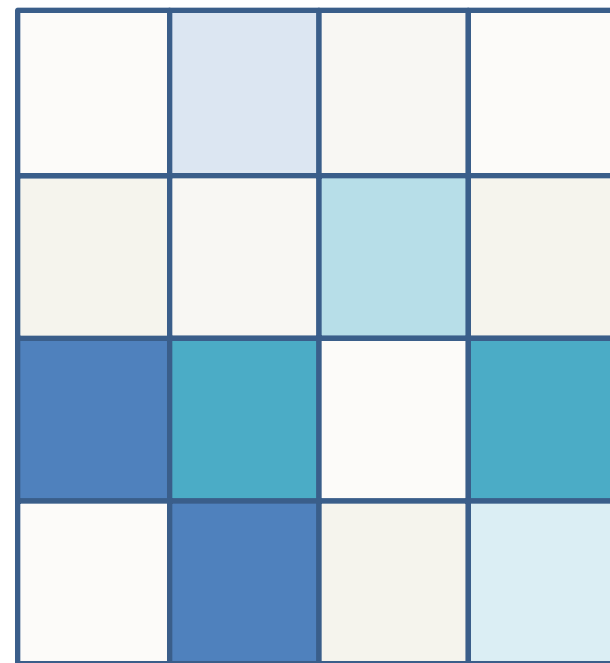




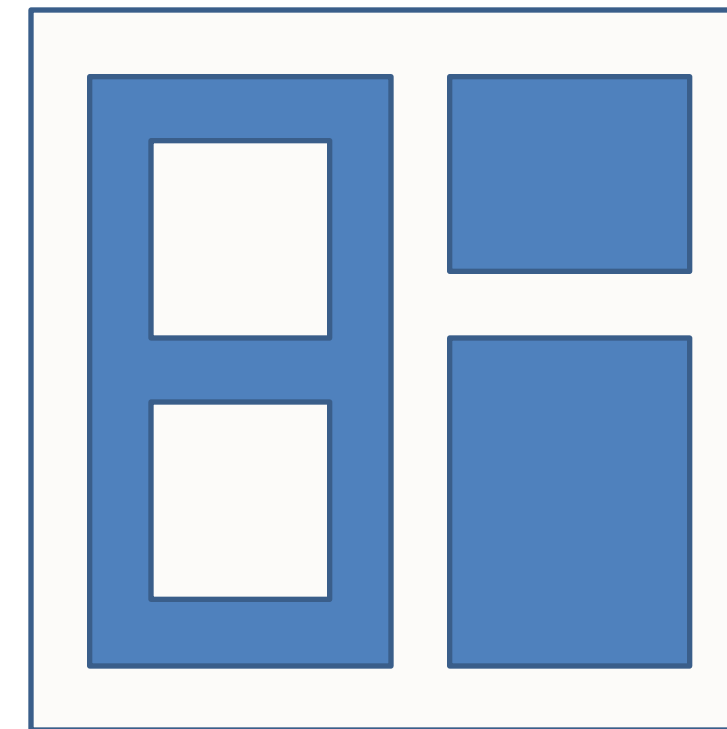
# Matrix Representations



Explicit  
(Node-Link)



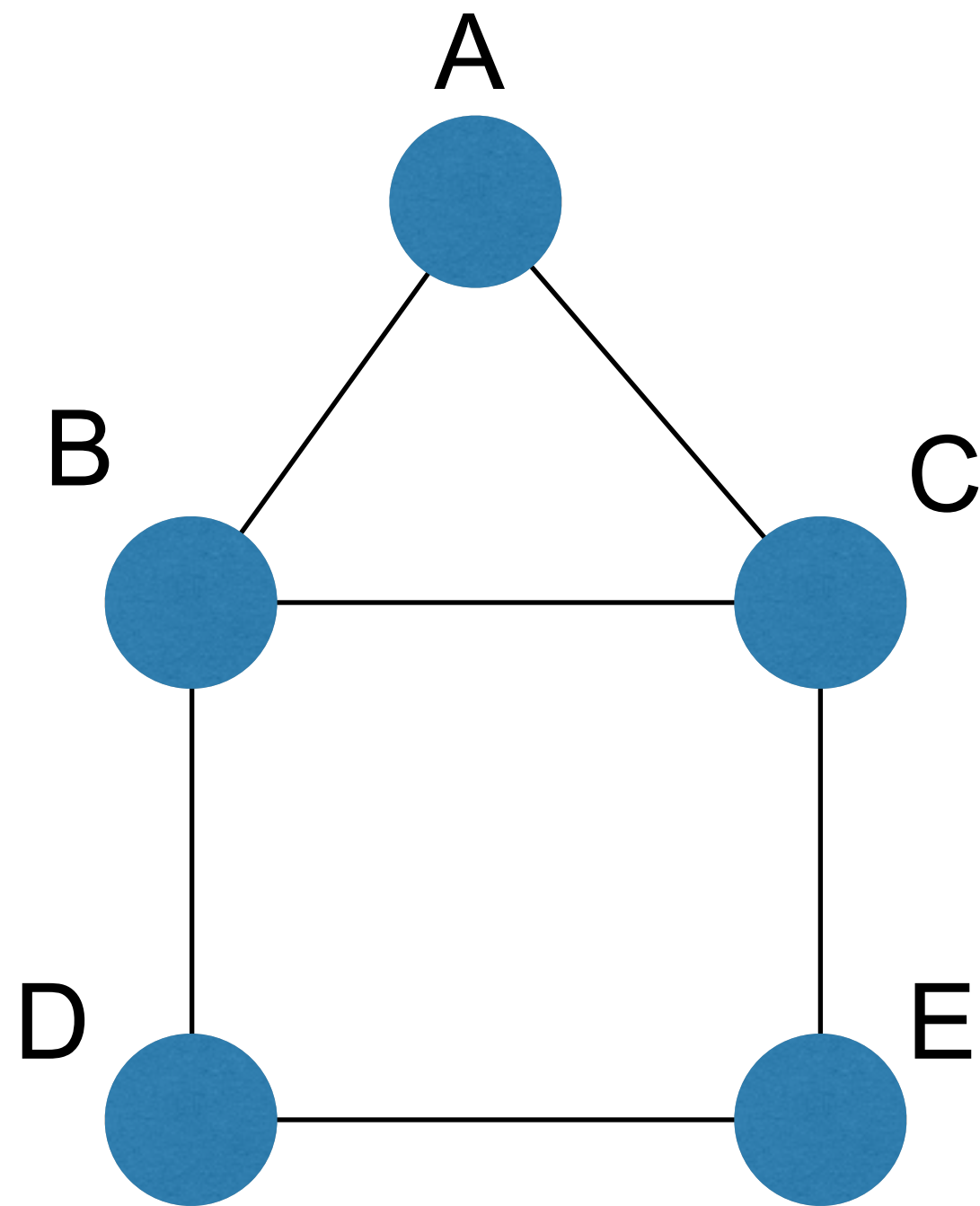
Matrix



Implicit

# Matrix Representations

Instead of node link diagram, use adjacency matrix

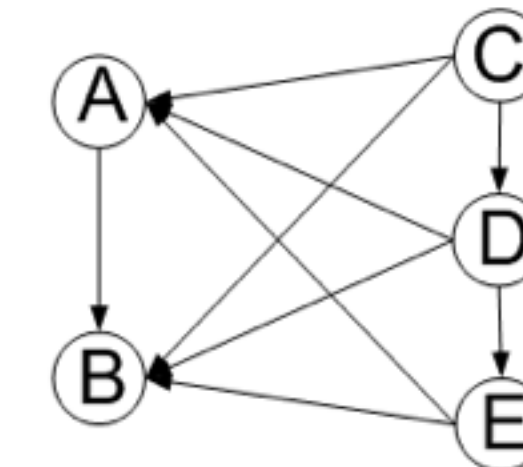
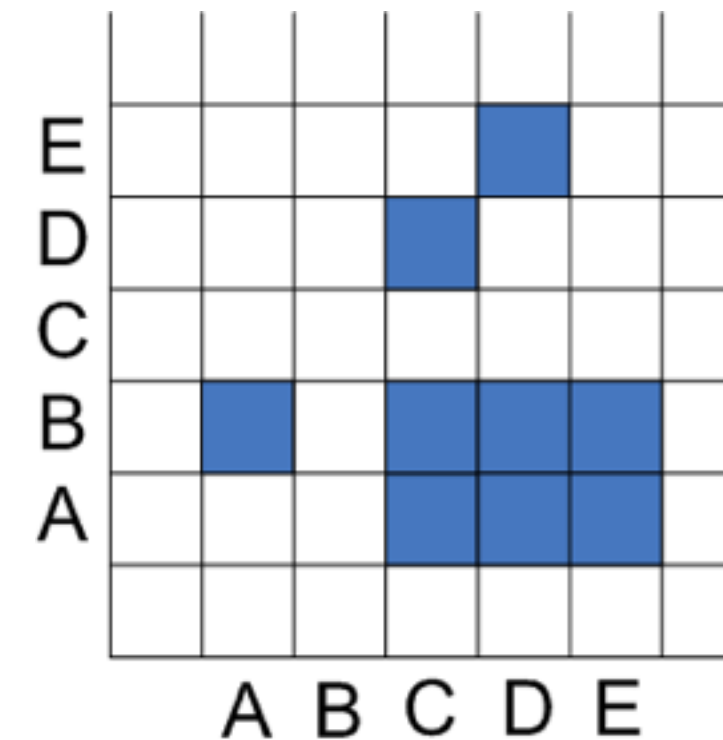
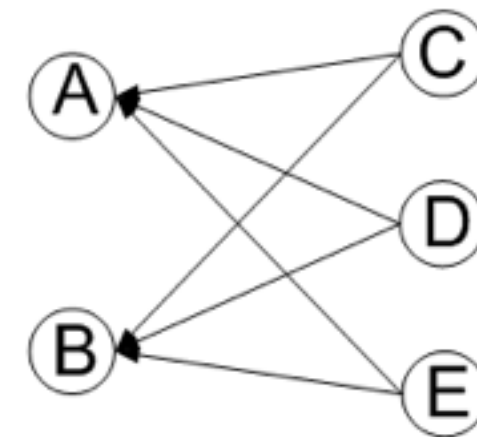
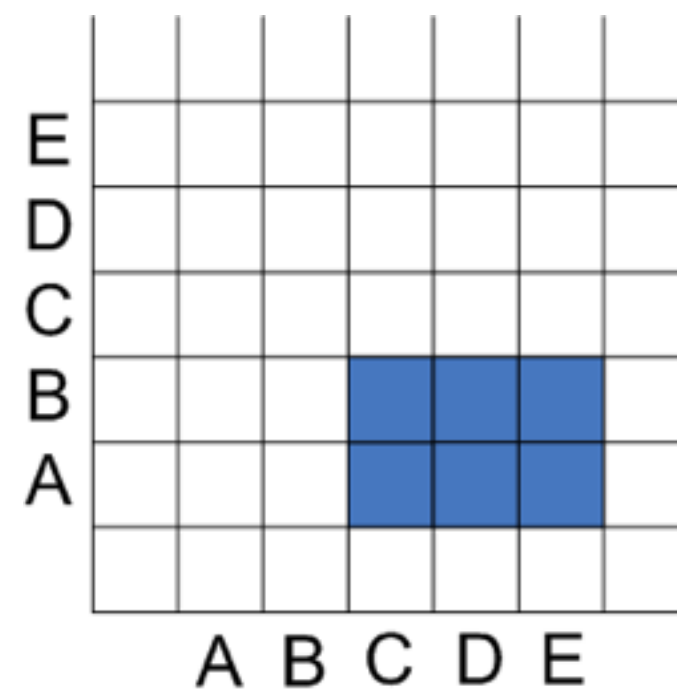
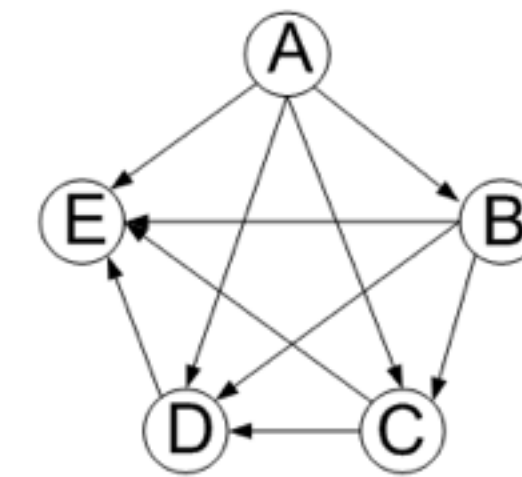
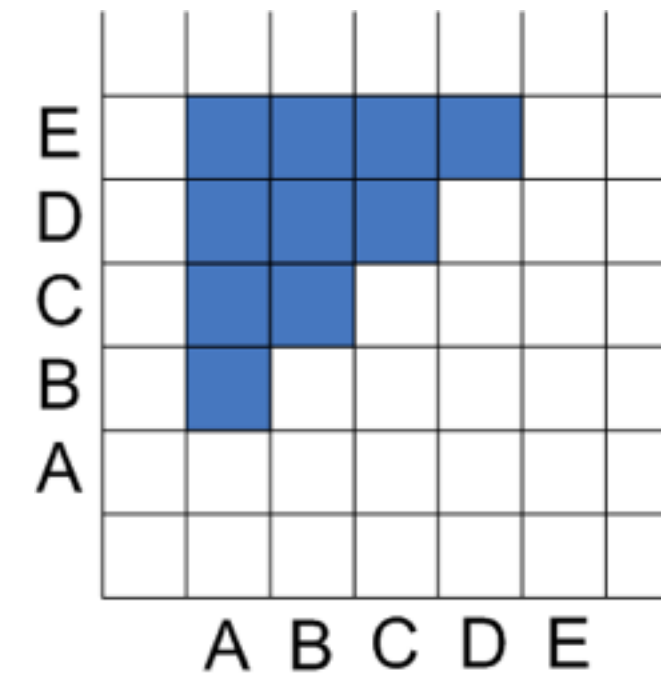
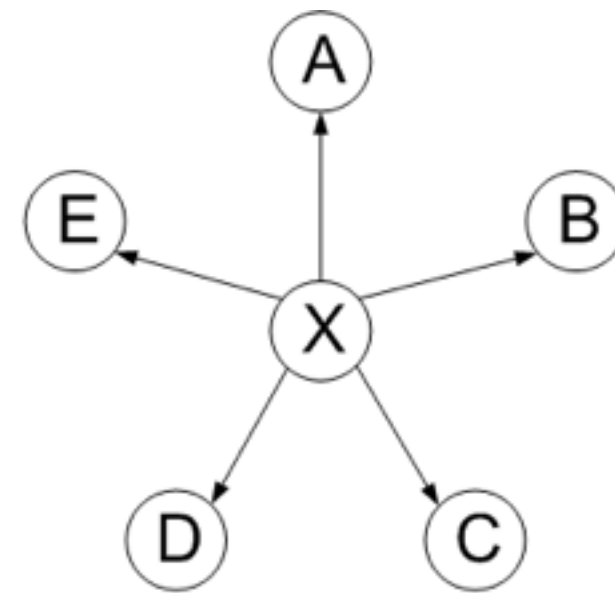
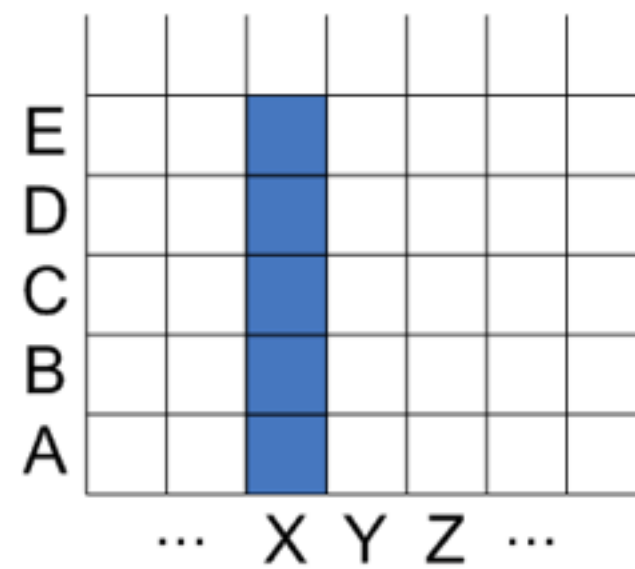


	A	B	C	D	E
A					
B					
C					
D					
E					



# Matrix Representations

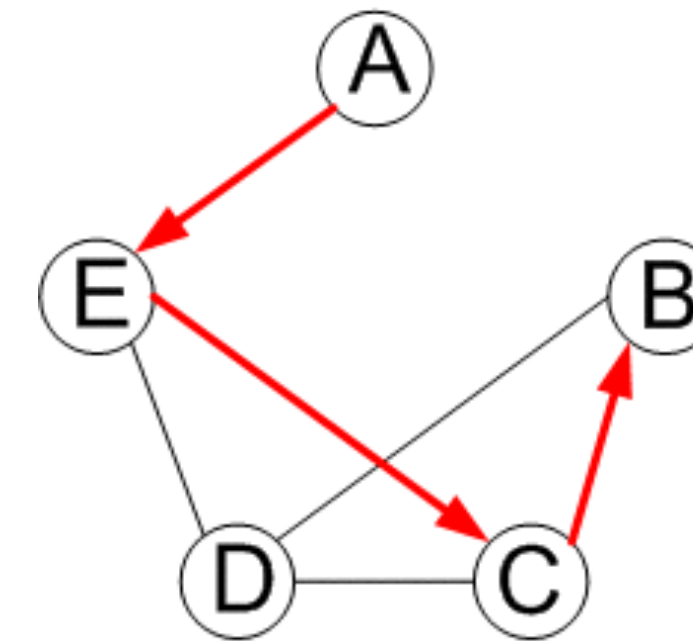
Examples:



# Matrix Representations

		TO							
		A	B	C	D	E	F	G	H
A			■	■					
B	■		■	■					
C		■				■			
D									■
E				■		■	■		
F					■			■	
G						■			■
H					■		■		

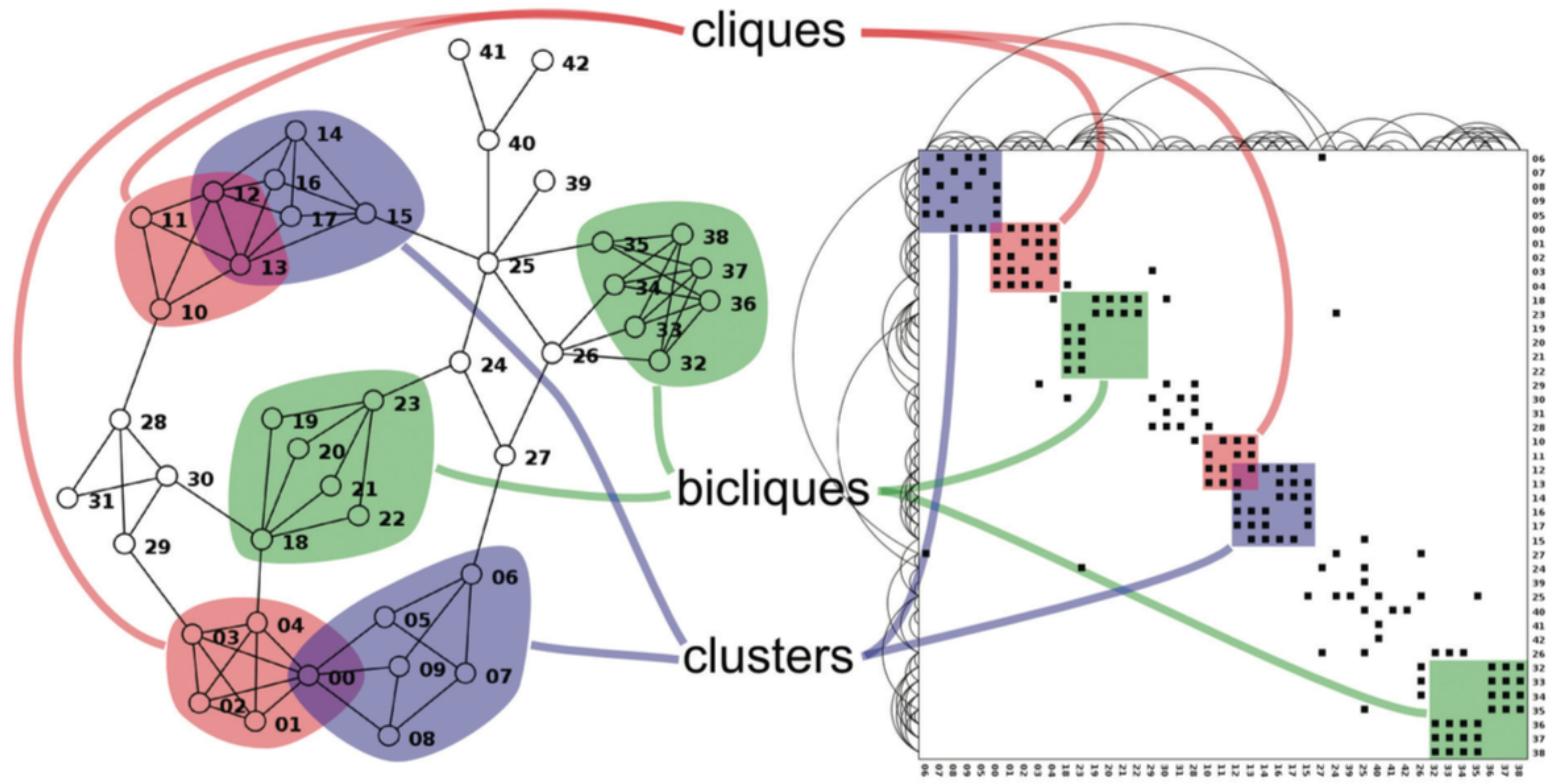
Well suited for  
neighborhood-related TBTs



E		■		■	■	
D			■	■		■
C			■		■	■
B				■		
A					■	
	A	B	C	D	E	

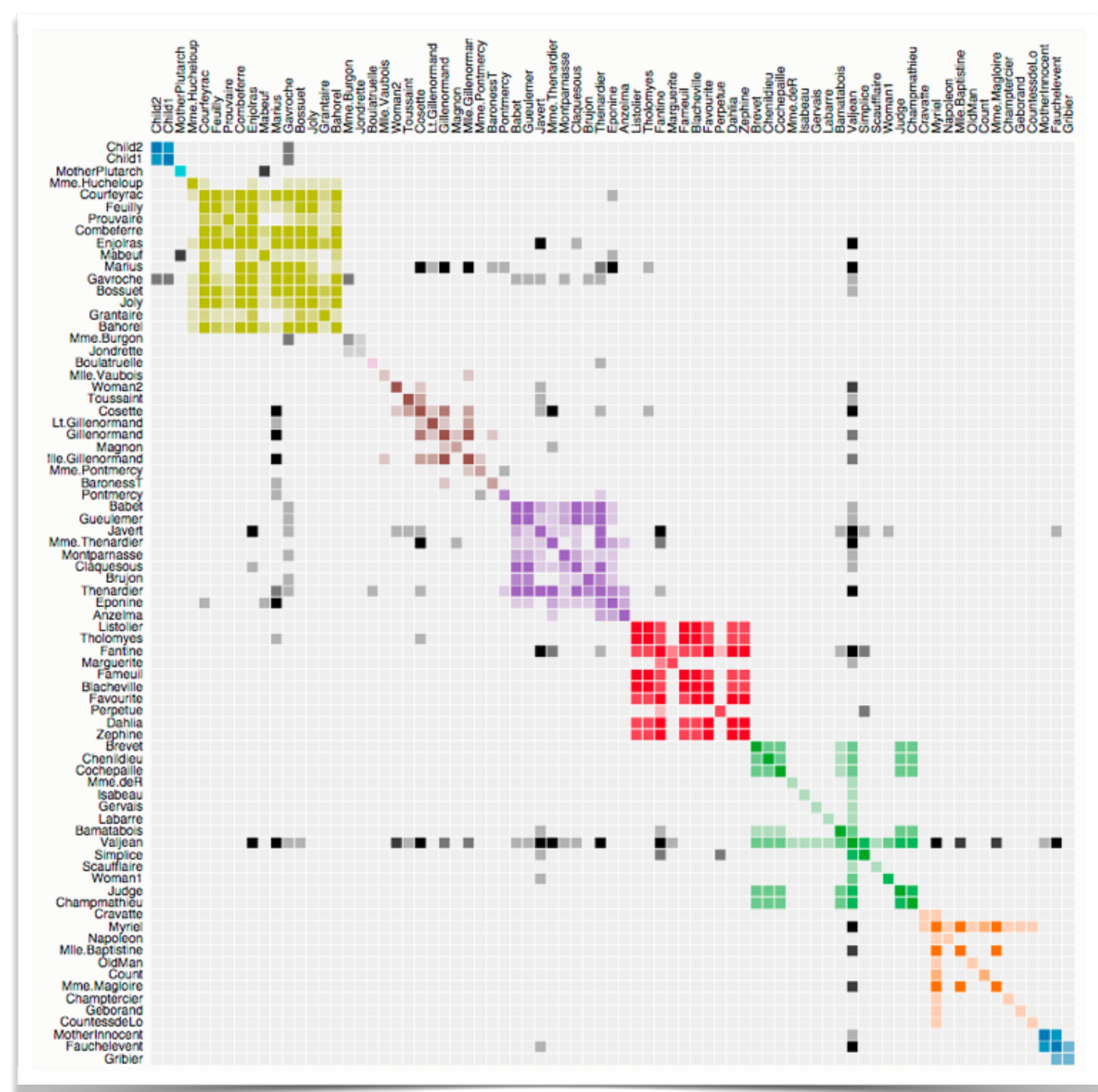
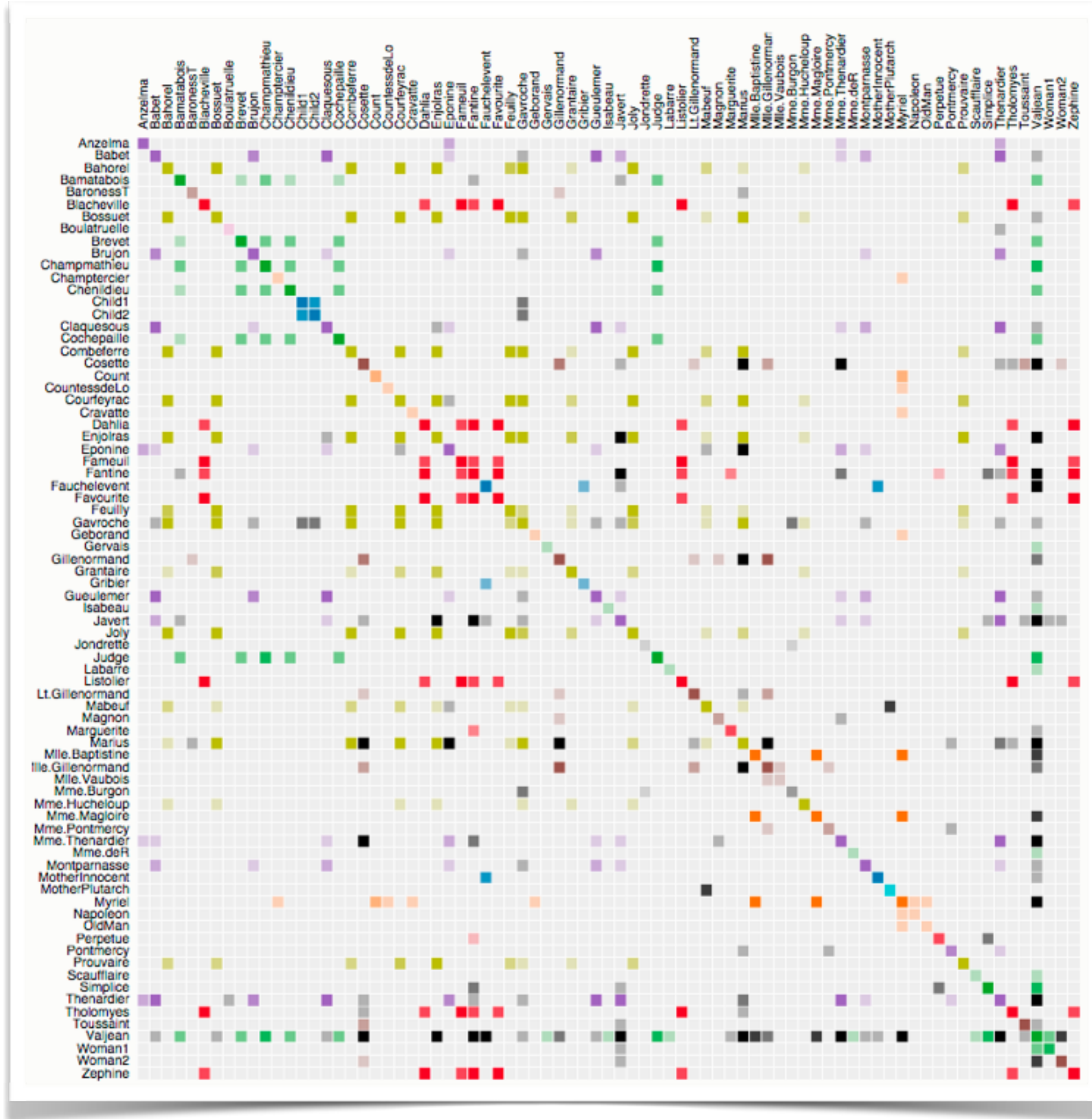
Not suited for  
path-related TBTs







# Order Critical!





# Matrix Representations

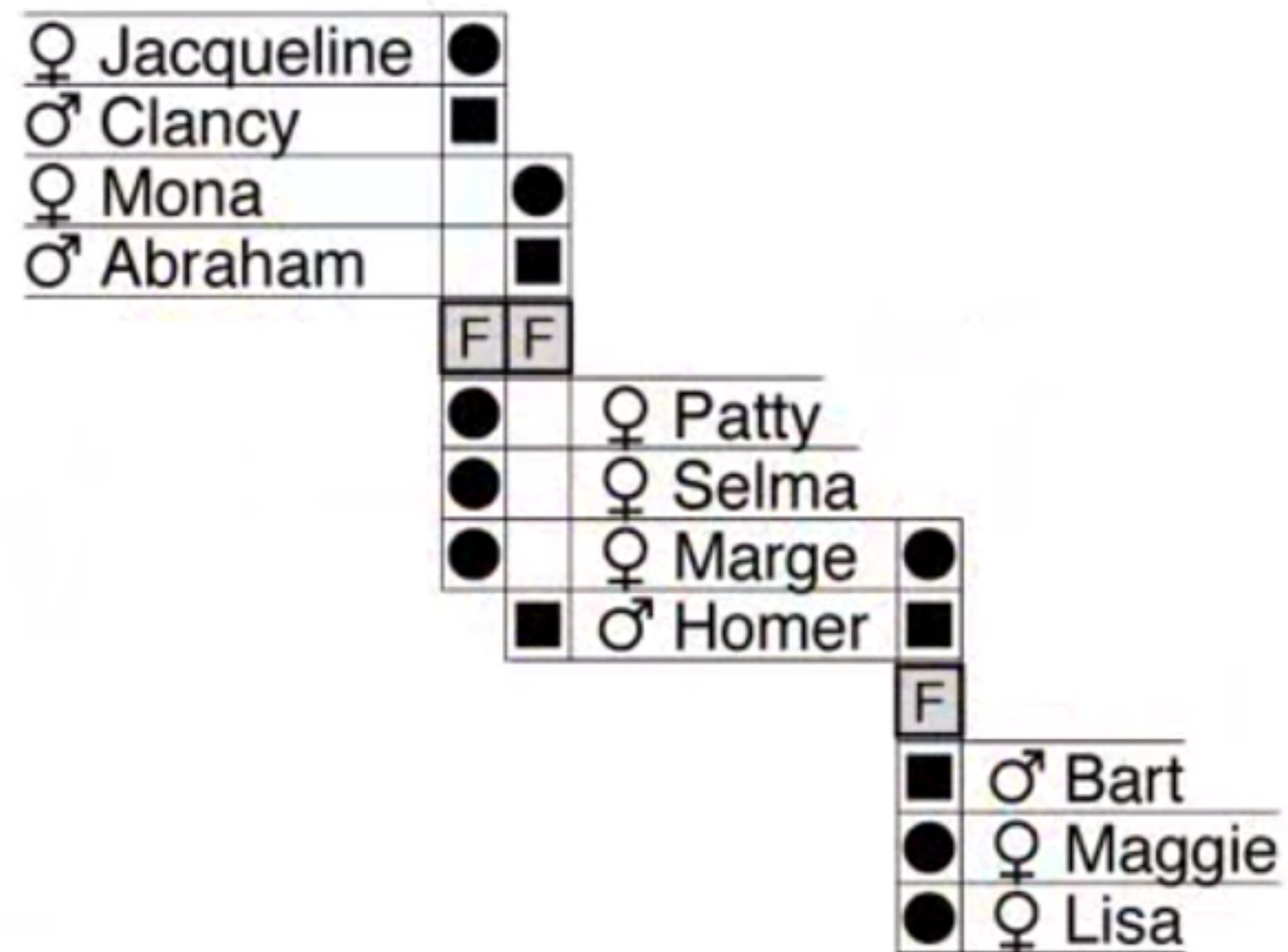
## Pros:

- can represent **all graph classes** except for hypergraphs
- puts **focus on the edge set**, not so much on the node set
- simple grid -> **no elaborate layout** or rendering needed
- well suited for **ABT on edges** via coloring of the matrix cells
- well suited for **neighborhood-related TBTs** via traversing rows/columns

## Cons:

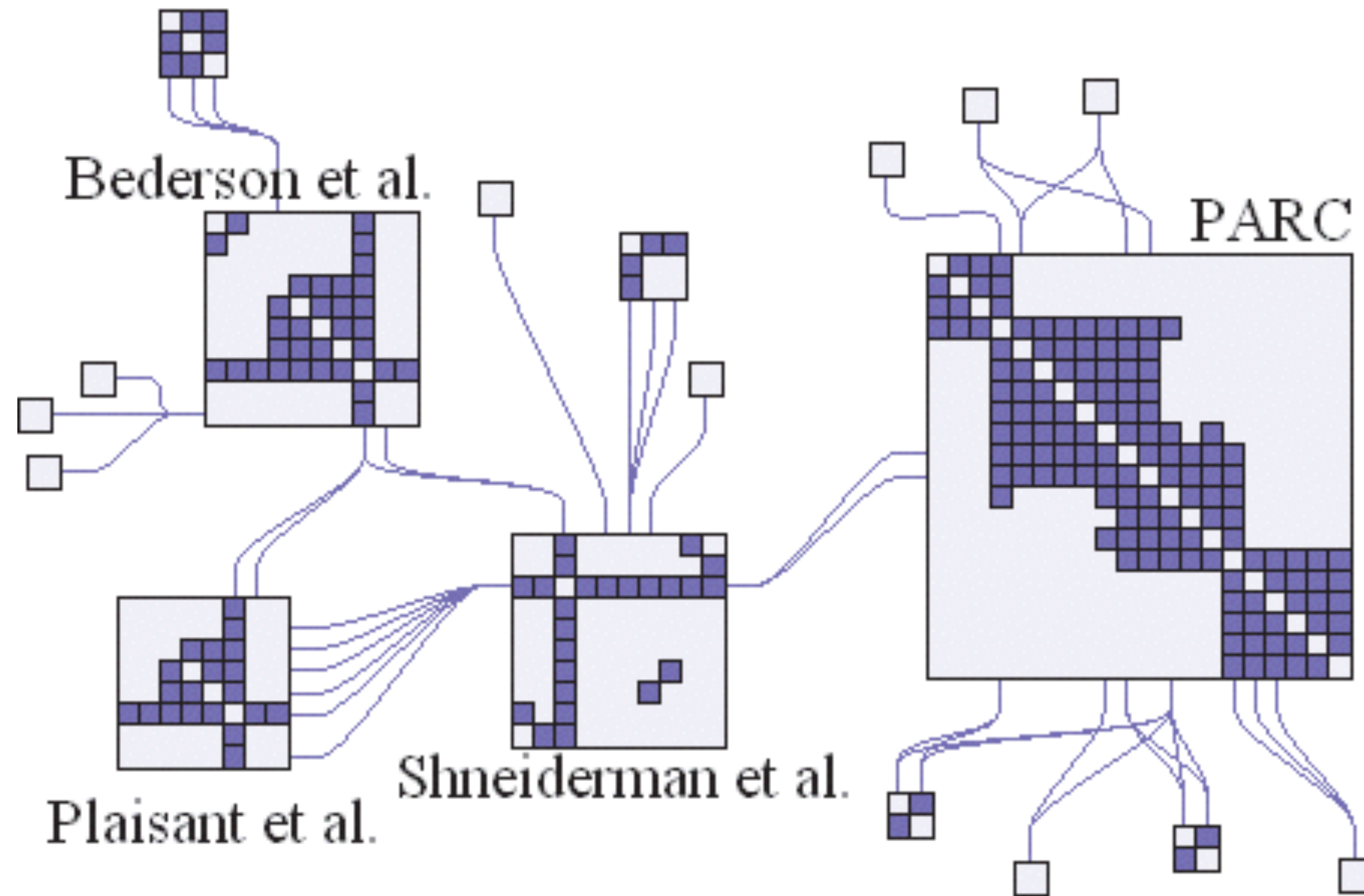
- quadratic screen space requirement (any possible edge takes up space)
- not suited for path-related TBTs

# Special Case: Genealogy

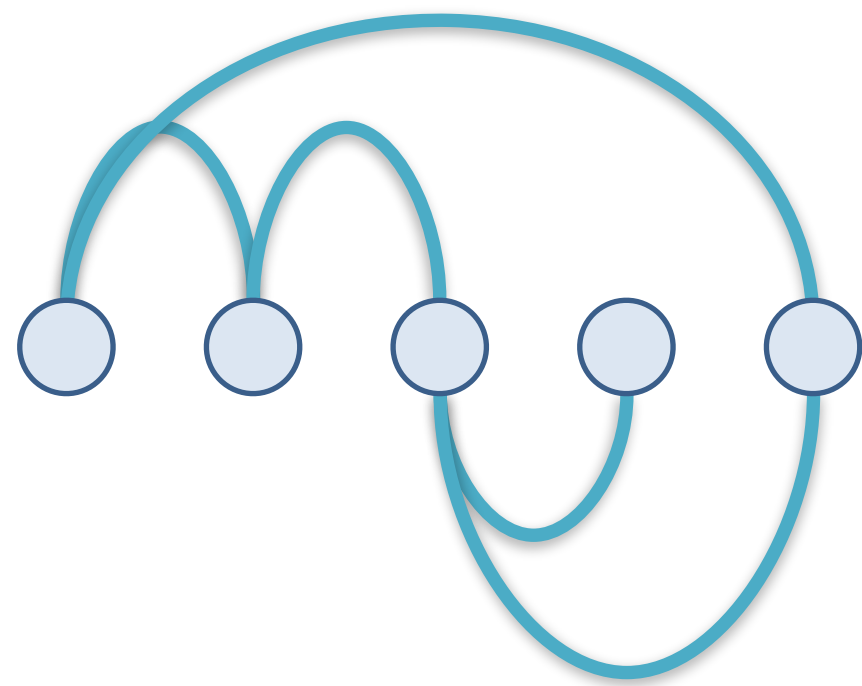




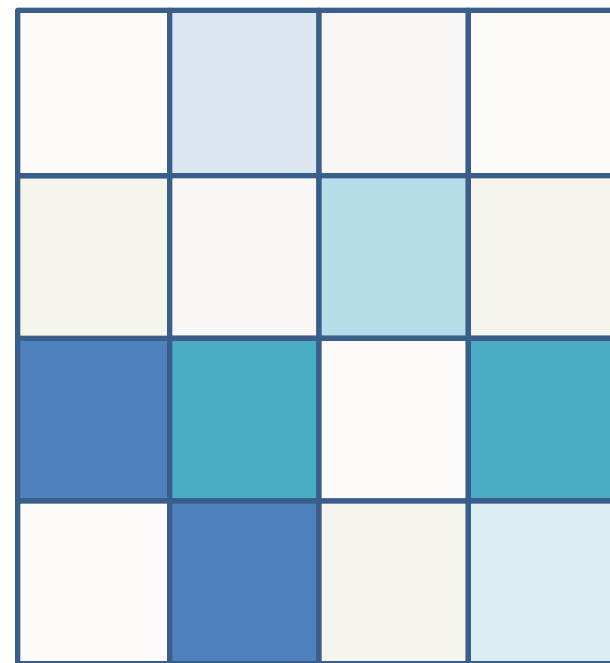
# Hybrid Explicit/Matrix



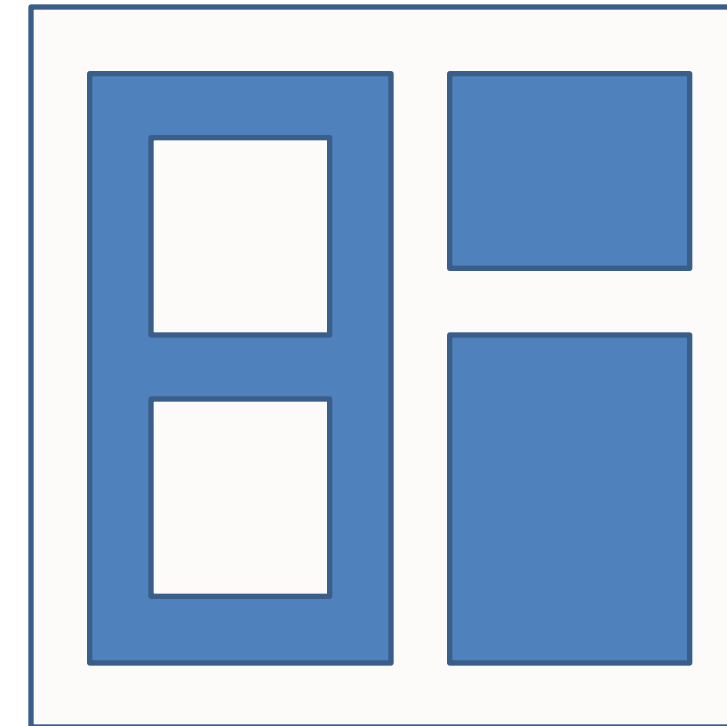
# Implicit Layouts



Explicit  
(Node-Link)



Matrix



Implicit



# Explicit vs. Implicit Tree Vis

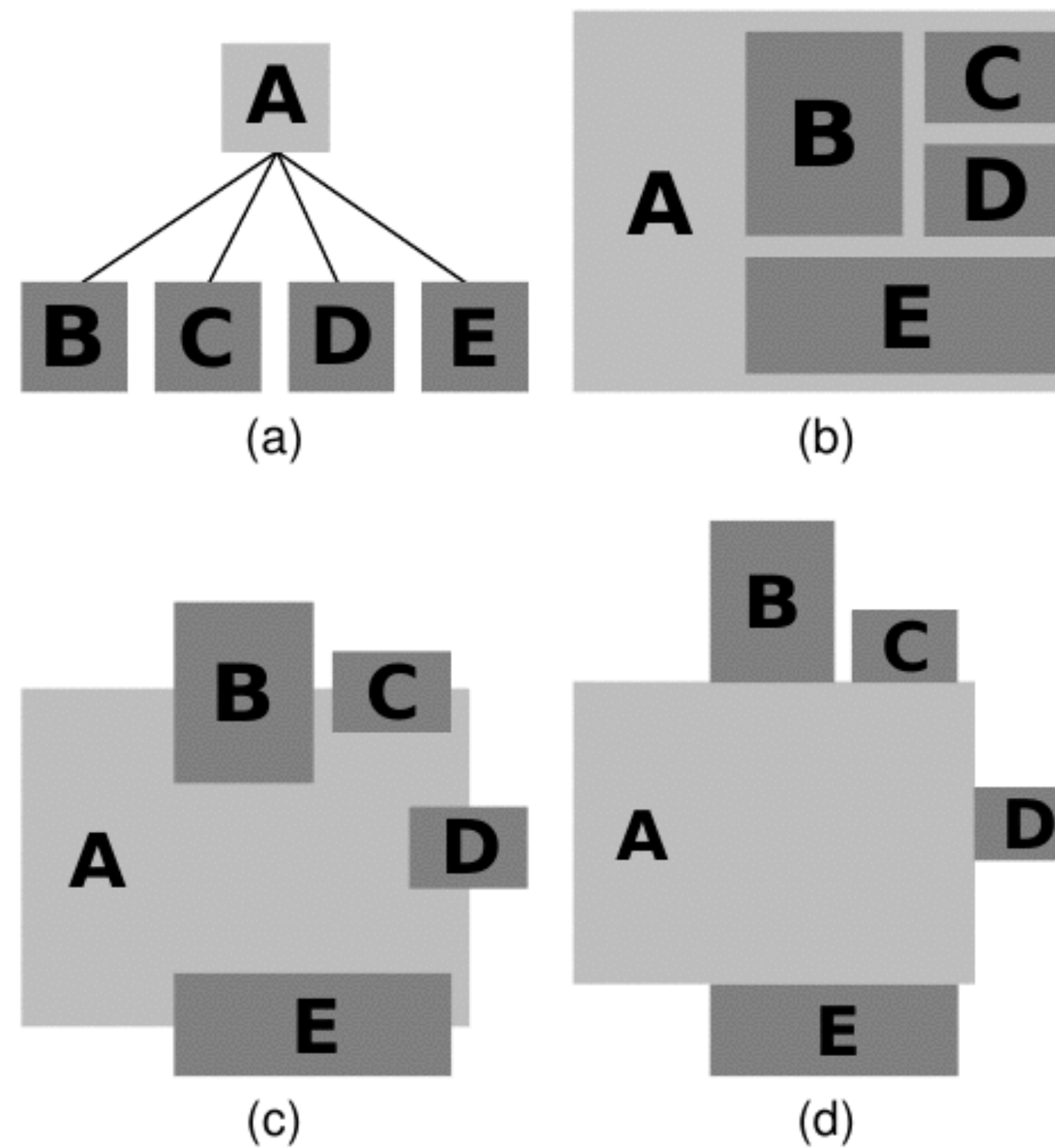
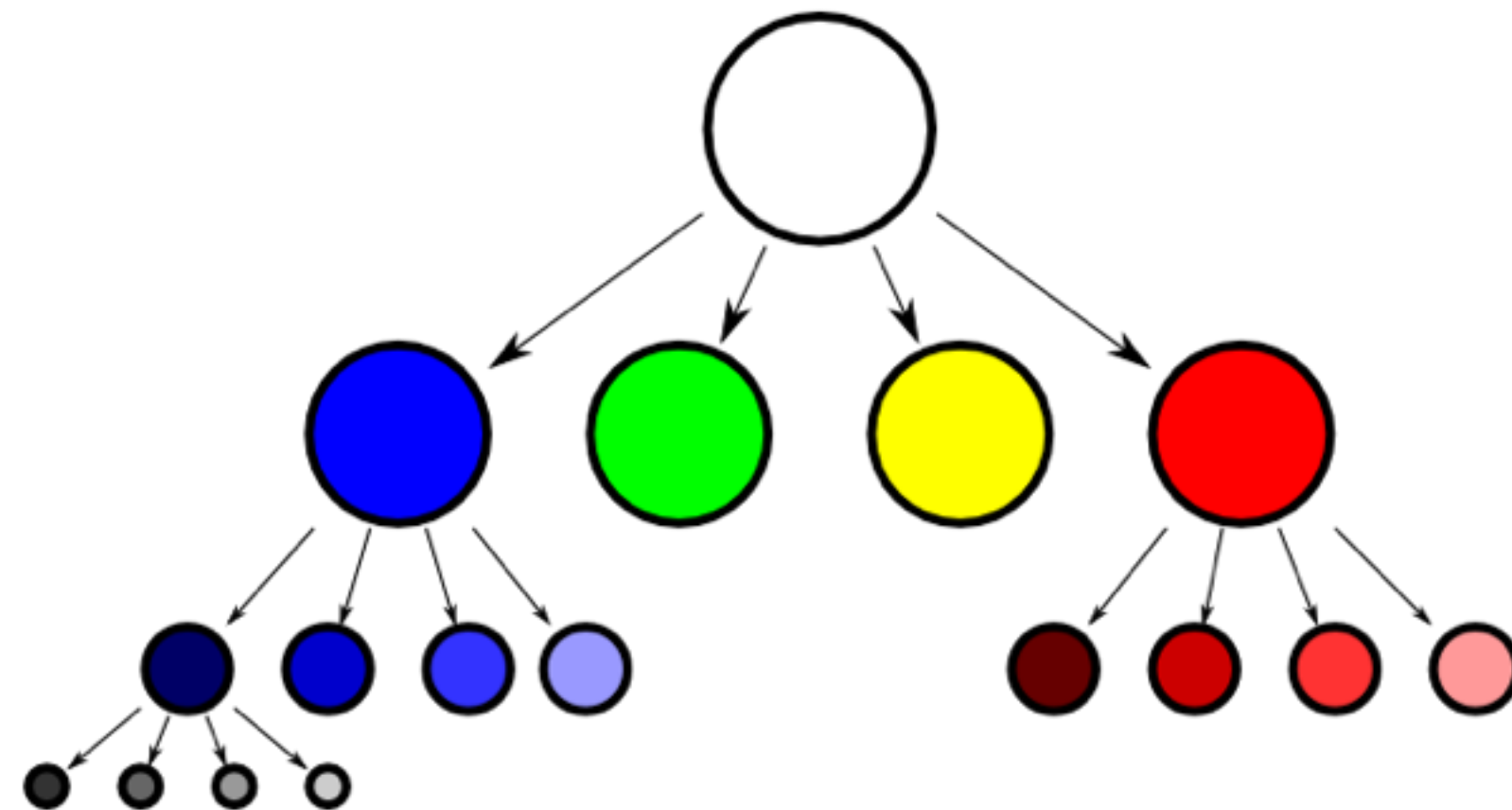
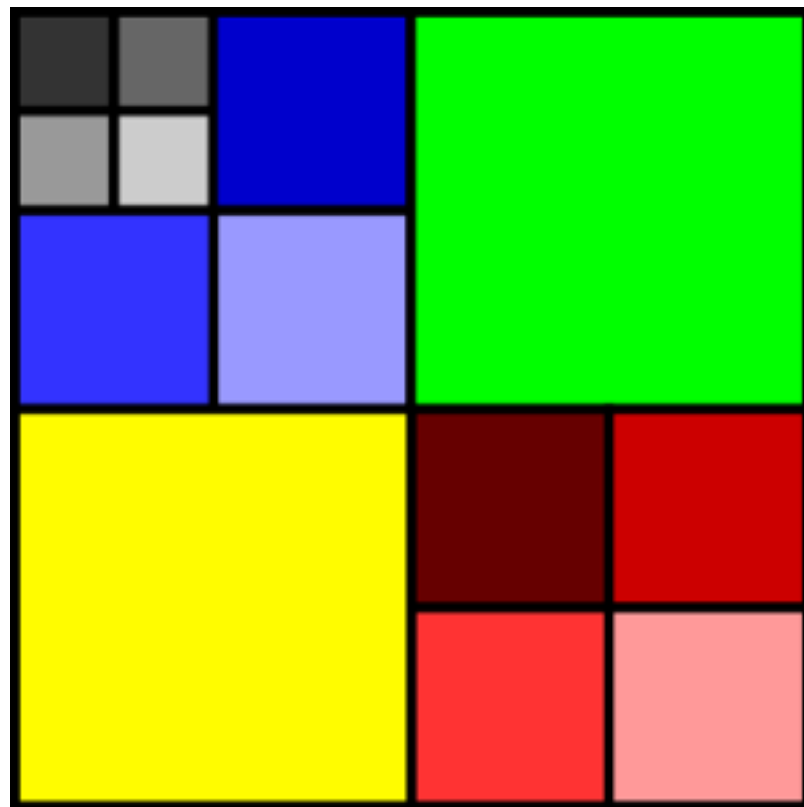
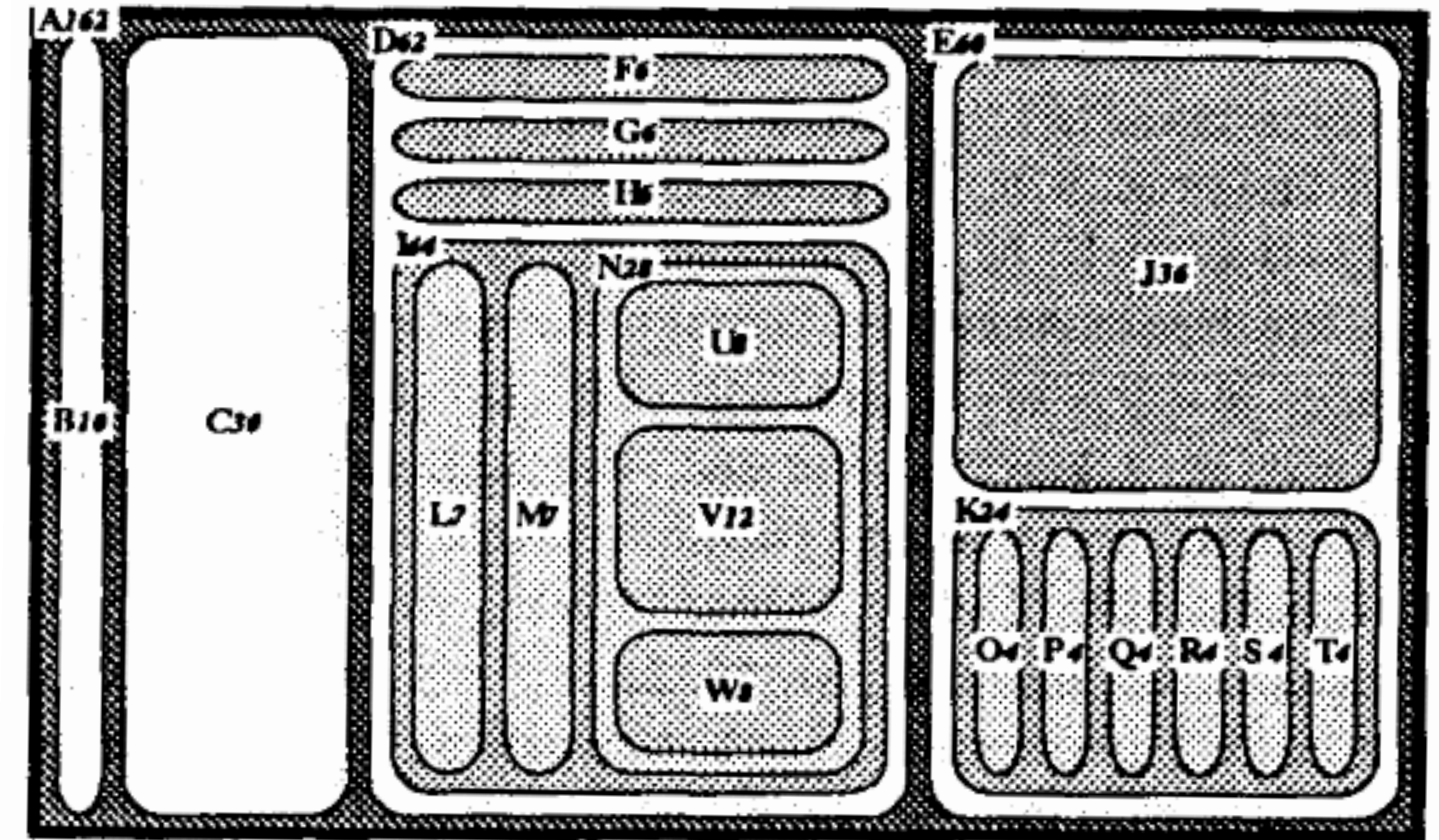
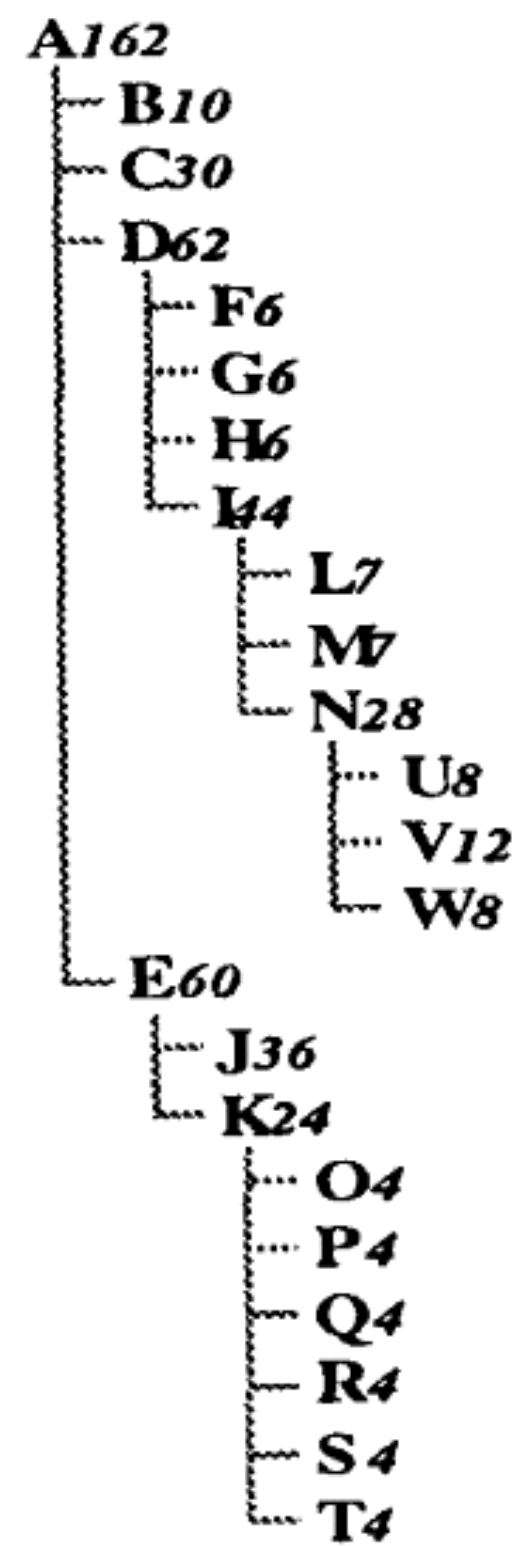


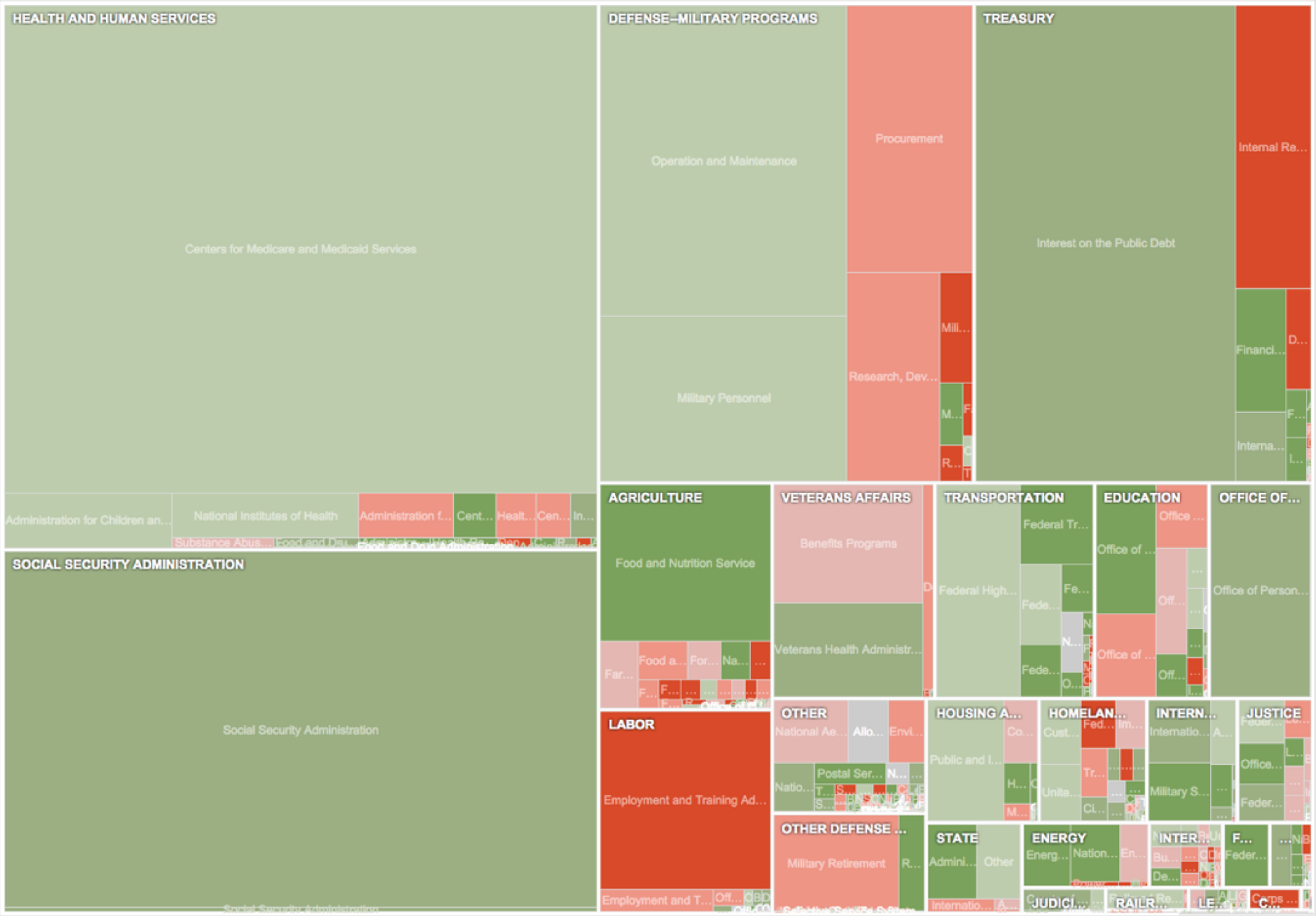
Fig. 2. (a) Explicit, node-link layout, (b) Implicit layout by inclusion, (c) Implicit Layout by overlap, (d) Implicit layout by adjacency.

# Tree Maps



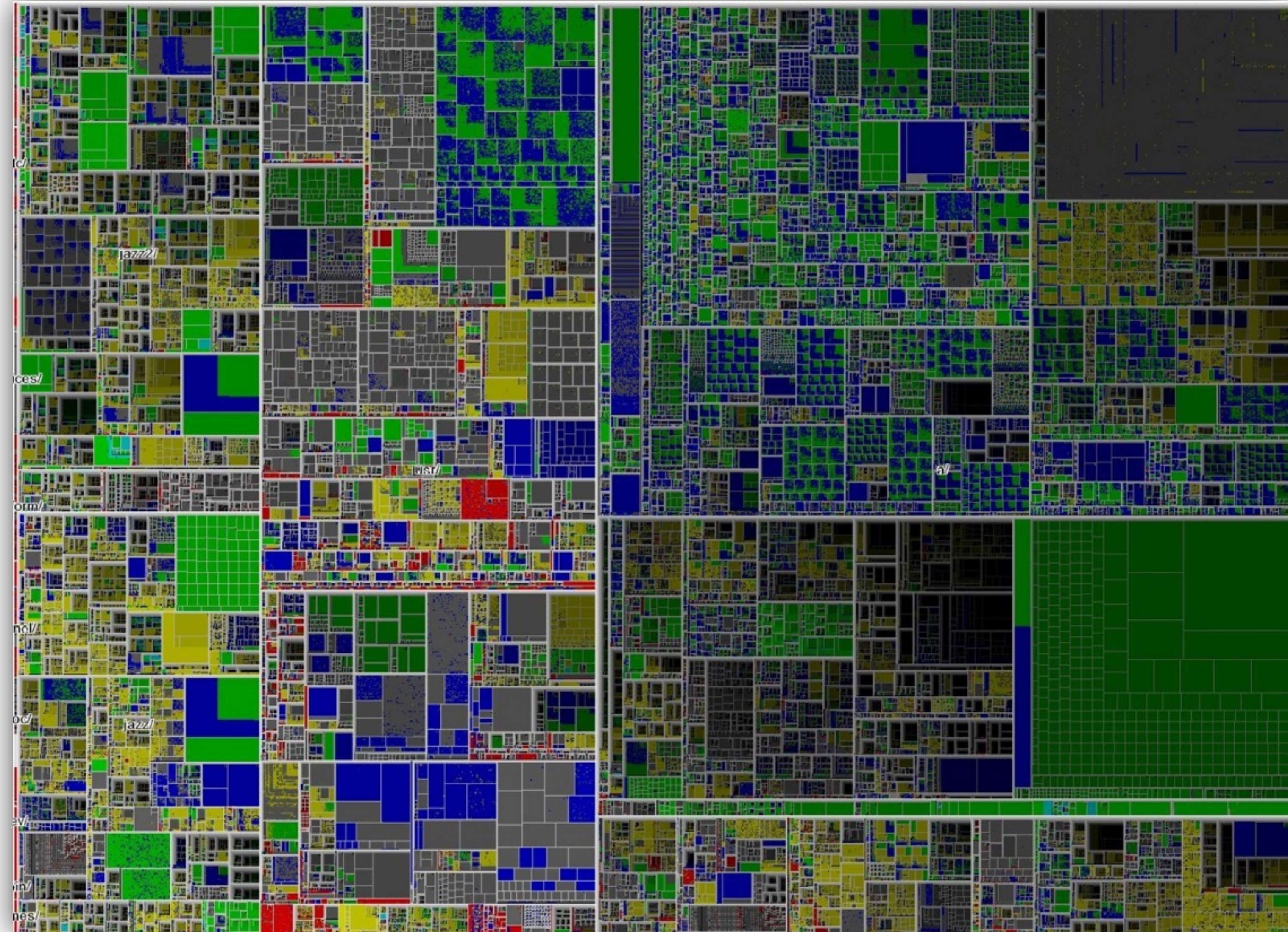


# Zoomable Treemap



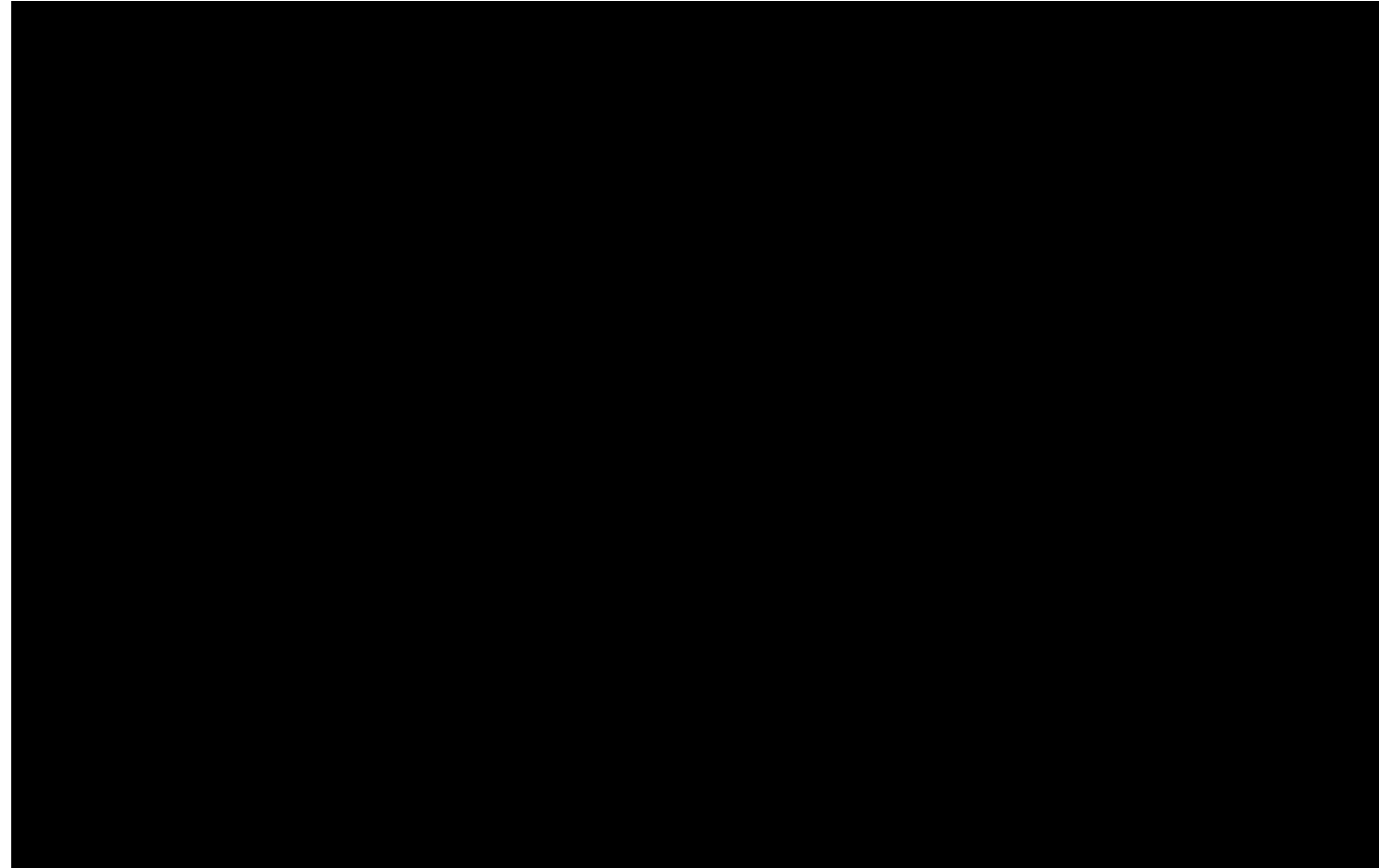


# Example: Interactive TreeMap of a Million Items





# Sunburst: Radial Layout



[Sunburst by John Stasko, Implementation in Caleydo by Christian Partl]









# Tree Visualization Reference




How to cite this site? [Check out other surveys!](#)

treevis.net - A Visual Bibliography of Tree Visualization 2.0 by Hans-Jörg Schulz

v.21-OCT-2014

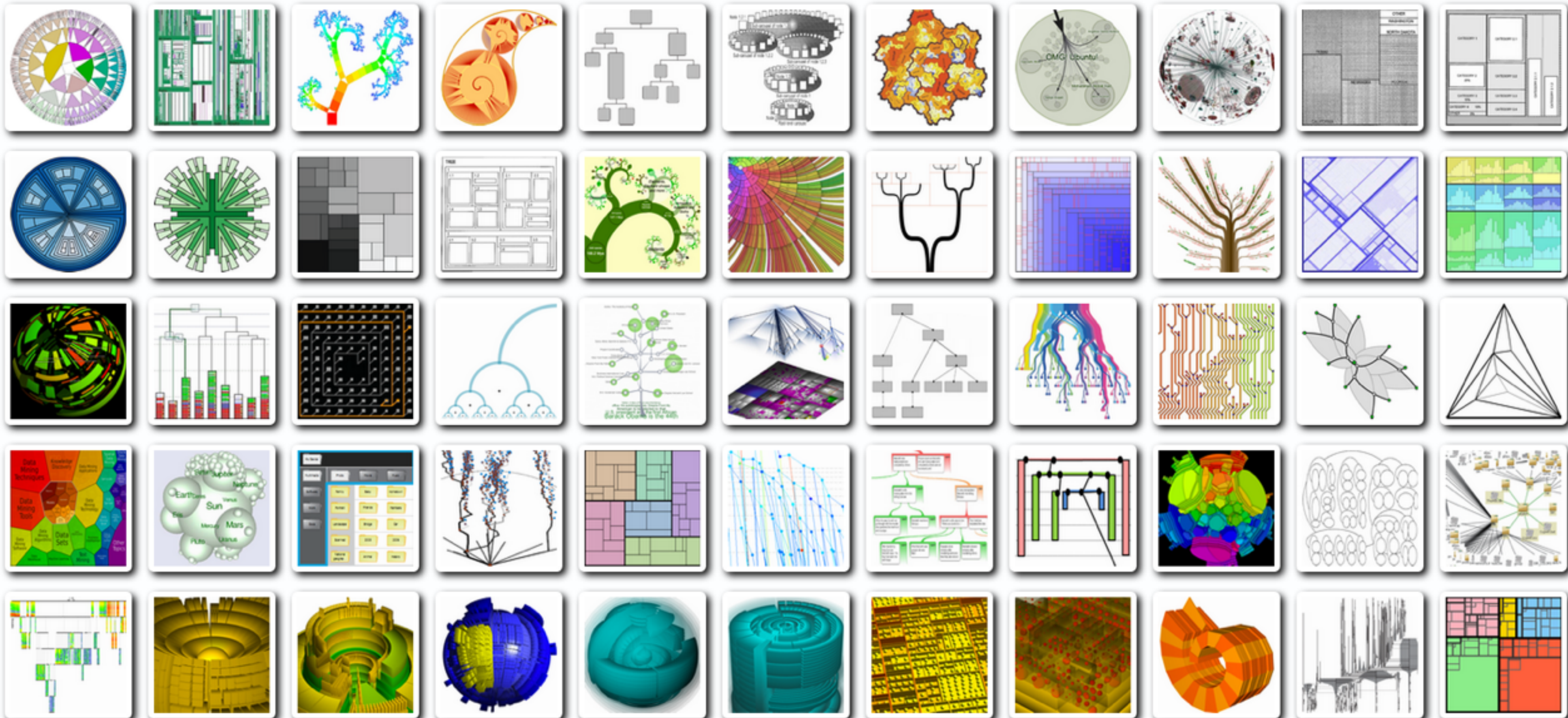
Dimensionality: All   

Representation: All   

Alignment: All   

Fulltext Search:  x

Techniques Shown: 277

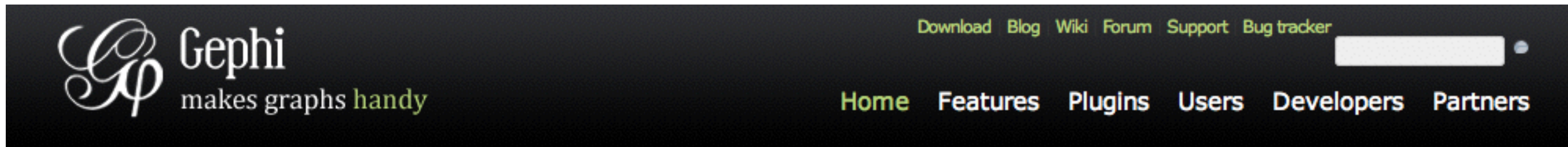




# Graph Tools & Applications

# Gephi

<http://gephi.org>



## The Open Graph Viz Platform

Gephi is a visualization and exploration platform for all kinds of networks and complex systems, dynamic and hierarchical graphs.

Runs on Windows, Linux and Mac OS X. Gephi is open-source and free.

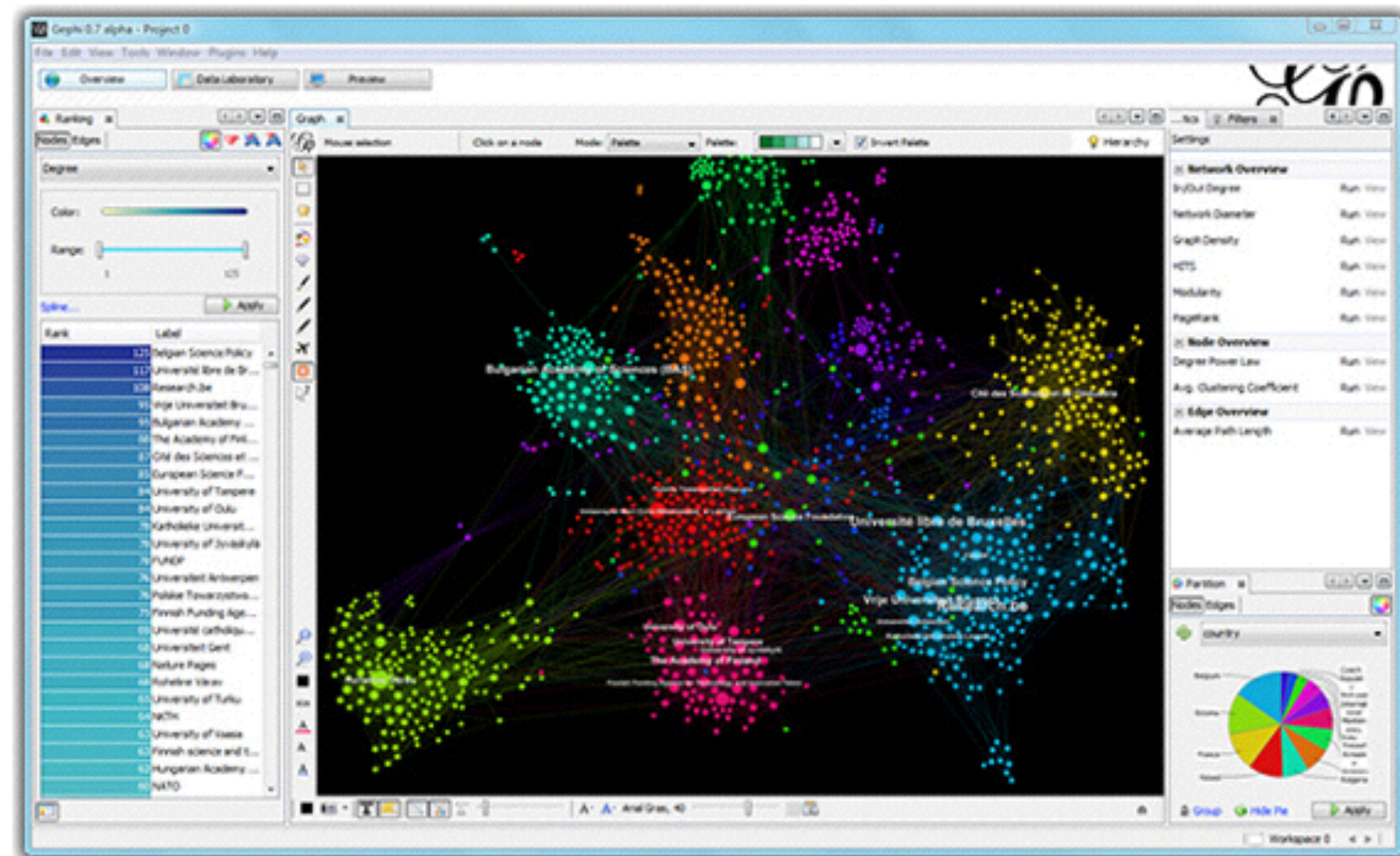
[Learn More on Gephi Platform »](#)



[Release Notes](#) | [System Requirements](#)

► [Features](#)  
► [Quick start](#)

► [Screenshots](#)  
► [Videos](#)



Gephi has been accepted again for Google Summer of Code! The program is the best way for students around the world to start contributing to an open-source project. Students, apply now for Gephi proposals. Come to the GSOC forum section and say Hi! to [this topic](#).

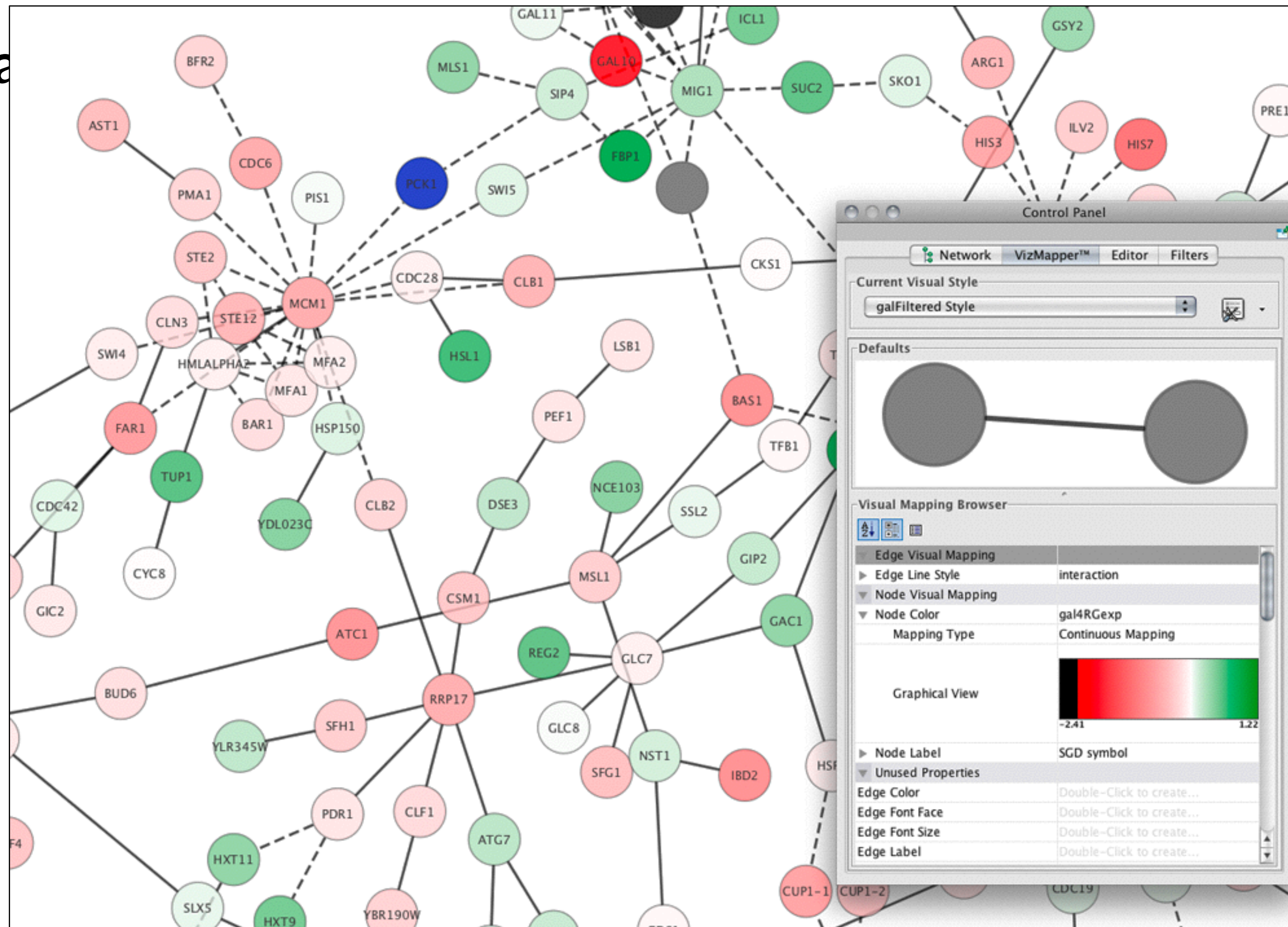
[Learn More »](#)



# Cytoscape

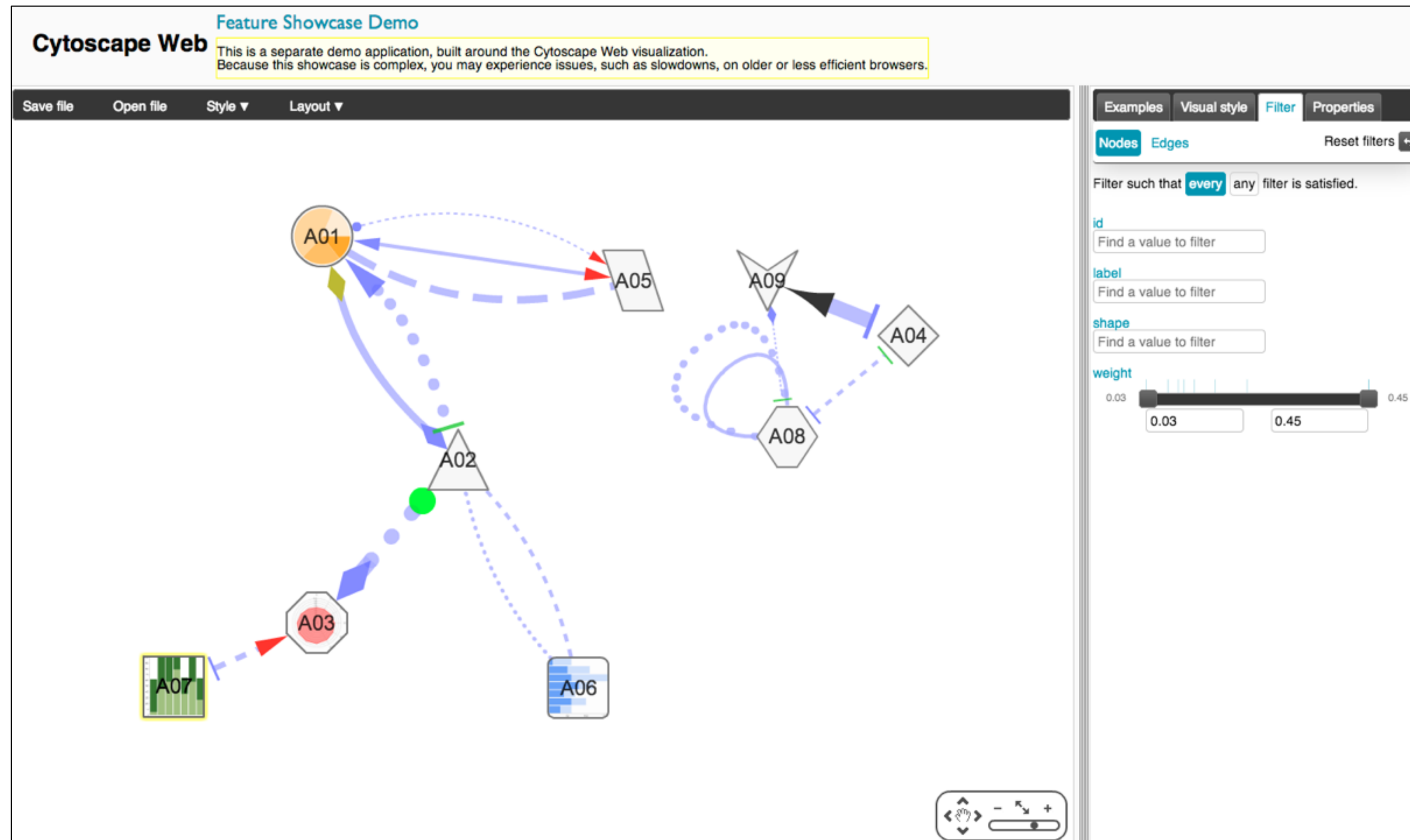
<http://www.cytoscape.org/>

Open source platform



# Cytoscape Web

<http://cytoscapeweb.cytoscape.org/>





# NetworkX

<https://networkx.github.io/>

## NetworkX

[NetworkX Home](#) | [Documentation](#) | [Download](#) | [Developer \(Github\)](#)

### High-productivity software for complex networks

NetworkX is a Python language software package for the creation, manipulation, and study of the structure, dynamics, and functions of complex networks.



#### [Documentation](#)

*all documentation*

#### [Examples](#)

*using the library*

#### [Reference](#)

*all functions and methods*

### Features

- Python language data structures for graphs, digraphs, and multigraphs.
- Nodes can be "anything" (e.g. text, images, XML records)
- Edges can hold arbitrary data (e.g. weights, time-series)
- Generators for classic graphs, random graphs, and synthetic networks
- Standard graph algorithms
- Network structure and analysis measures
- Open source [BSD license](#)
- Well tested: more than 1800 unit tests, >90% code coverage
- Additional benefits from Python: fast prototyping, easy to teach, multi-platform

#### Versions

#### Latest Release

1.8.1 - 4 August 2013

[downloads](#) | [docs](#) | [pdf](#)

#### Development

1.9dev

[github](#) | [docs](#) | [pdf](#)

build passing

coverage 83%

#### Contact

[Mailing list](#)  
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[Developer guide](#)

